Coimisiún na Scrúduithe Stáit
State Examinations Commission

Leaving Certificate 2020

Marking Scheme

Mathematics

Higher Level

Note to teachers and students on the use of published marking schemes

Marking schemes published by the State Examinations Commission are not intended to be standalone documents. They are an essential resource for examiners who receive training in the correct interpretation and application of the scheme. This training involves, among other things, marking samples of student work and discussing the marks awarded, so as to clarify the correct application of the scheme. The work of examiners is subsequently monitored by Advising Examiners to ensure consistent and accurate application of the marking scheme. This process is overseen by the Chief Examiner, usually assisted by a Chief Advising Examiner. The Chief Examiner is the final authority regarding whether or not the marking scheme has been correctly applied to any piece of candidate work.

Marking schemes are working documents. While a draft marking scheme is prepared in advance of the examination, the scheme is not finalised until examiners have applied it to candidates' work and the feedback from all examiners has been collated and considered in light of the full range of responses of candidates, the overall level of difficulty of the examination and the need to maintain consistency in standards from year to year. This published document contains the finalised scheme, as it was applied to all candidates' work.

In the case of marking schemes that include model solutions or answers, it should be noted that these are not intended to be exhaustive. Variations and alternatives may also be acceptable. Examiners must consider all answers on their merits, and will have consulted with their Advising Examiners when in doubt.

Future Marking Schemes

Assumptions about future marking schemes on the basis of past schemes should be avoided. While the underlying assessment principles remain the same, the details of the marking of a particular type of question may change in the context of the contribution of that question to the overall examination in a given year. The Chief Examiner in any given year has the responsibility to determine how best to ensure the fair and accurate assessment of candidates' work and to ensure consistency in the standard of the assessment from year to year. Accordingly, aspects of the structure, detail and application of the marking scheme for a particular examination are subject to change from one year to the next without notice.

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Marking Scheme – Paper 1, Section A and Section B

Structure of the marking scheme

Candidate responses are marked according to different scales, depending on the types of response anticipated. Scales labelled A divide candidate responses into two categories (correct and incorrect). Scales labelled B divide responses into three categories (correct, partially correct, and incorrect), and so on. The scales and the marks that they generate are summarised in this table:

Scale label	A	B	C	D	E
No of categories	2	3	4	5	6
5 mark scales	0, 5	0, 2, 5	0, 3, 4, 5		
10 mark scales	0, 10	0, 5, 10	0, 4, 8, 10	0, 3, 5, 8, 10	
15 mark scales	0, 15	0, 7, 15	0, 5, 10, 15	0, 4, 7, 11, 15	
20 mark scales	0, 20	0, 10, 20	0, 7, 13, 20	0, 5, 10, 15, 20	
25 mark scales	0, 25	0, 12, 25	0, 8, 17, 25	0, 6, 12, 19, 25	0, 5, 10, 15, 20, 25

A general descriptor of each point on each scale is given below. More specific directions in relation to interpreting the scales in the context of each question are given in the scheme, where necessary.

Marking scales – level descriptors

A-scales (two categories)

- incorrect response
- correct response

B-scales (three categories)

- response of no substantial merit
- partially correct response
- correct response

C-scales (four categories)

- response of no substantial merit
- response with some merit
- almost correct response
- correct response

D-scales (five categories)

- response of no substantial merit
- response with some merit
- response about half-right
- almost correct response
- correct response

E-scales (six categories)

- response of no substantial merit
- response with some merit
- response almost half-right
- response more than half-right
- almost correct response
- correct response

NOTE: In certain cases, typically involving incorrect rounding, omission of units, a misreading that does not oversimplify the work or an arithmetical error that does not oversimplify the work, a mark that is one mark below the full-credit mark may also be awarded.
Rounding and units penalty to be applied only once in each section (a), (b), (c) etc.
Throughout the scheme indicate by use of * where an arithmetic error occurs.

Summary of mark allocations and scales to be applied

Section A

Question 1

(a)(i)	10C
(a)(ii)	5C
(a)(iii)	5C
(b)	5D

Question 2

(a)	5D
(b) (i)	5C
(b) (ii)	15D

Question 3

(a)	10C
(b)(i)	5C
(b)(ii)	10D

Question 4

(a)	15D
(b)	5D
(c)	5C

Question 5

(a)	15C
(b)	10D

Question 6

(a)	15C
(b)(i)	5C
(b)(ii)	5D

Section B

Question 7 (50 Marks)

(a)(i)	10C
(a)(ii)	5C
(b)(i)	5C
(b)(ii)	5C
(b)(iii)	5C
(c)	5C
(d)	15D

Question 8 (45 Marks)

(a)(i)	5C
(a)(ii)	10C
(a)(iii)	15D
(a)(iv)	5C
(b)	10D

Question 9 (55 Marks)

(a)(i)	10C
(a)(ii)	10C
(b)	10D
(c)	10C
(d)	5C
(e)	10C

Palette of annotations available to examiners

Symbol	Name	Meaning in the body of the work	Meaning when used in the right margin
✔	Tick	Work of relevance	The work presented in the body of the script merits full credit
✖	Cross	Incorrect work (distinct from an error)	The work presented in the body of the script merits 0 credit
$\boxed{*}$	Star	Rounding or Unit or Arithmetic error Misreading	
$\boxed{\sim\sim\sim}$	Horizontal wavy	Error	
✔$_l$	Tick L		The work presented in the body of the script merits low partial credit
✔$_m$	Tick M		The work presented in the body of the script merits mid partial credit (or partial credit)
✔$_h$	Tick H		The work presented in the body of the script merits high partial credit
$\boxed{F*}$	F star		The work presented in the body of the script merits Full Credit (– 1)
[	Left Bracket		Another version of this solution is presented elsewhere and it merits equal or higher credit
⌇	Vertical wavy	No work on this page (portion of the page)	
$\boxed{o}$	Oversimplify	The candidate has oversimplified the work	

Note: Where work of substance is presented in the body of the script, the annotation on the right margin should reflect a combination of annotations in the work

e.g. In a **C scale** where * and $\boxed{\sim\sim\sim}$ and $\boxed{\sim\sim\sim}$ appear in the body of the work then ✔$_l$ should be placed in the right margin.

In the case of a **D scale** with the same level of annotation then ✔$_m$ should be placed in the right margin.

A ✔ in the body of the work may sometimes be used to indicate where a portion of the work presented has value and has merited one of the levels of credit described in the marking scheme. The level of credit is them indicated in the right margin.

Detailed marking notes

Model Solutions & Marking Notes

Note: The model solutions for each question are not intended to be exhaustive – there may be other correct solutions. Any Examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his / her Advising Examiner.

Q1	**Model Solution – 25 Marks**	**Marking Notes**
(a) **(i)**	$f(-3)=0$ $f(-3)=-3^2+5(-3)+p=0$ $9-15+p=0$ $p=6$ **Or** $x^2+5x+p=(x+3)(x+a)$ $=x^2+x(a+3)+3a$ $a+3=5$ $a=2$ $p=3a$ $p=6$ **Or** $x+2$ $x+3 \;)\; x^2+5x+p$ x^2+2x $3x+p$ $3x+6$ $p-6$ $p-6=0$ $p=6$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Demonstrates understanding of $x+3$ as factor or – 3 as root e.g. $(x+3)$, $f(-3)$ *High Partial Credit:* Relevant equation in p (with p as only unknown)

(a) (ii)	$x^2 + 5x + p = (x - \alpha)(x - \alpha - 3)$ $= x^2 + x(-\alpha - \alpha - 3) + \alpha^2 + 3\alpha$ $-2\alpha - 3 = 5$ $\alpha = -4$ $p = 16 - 12$ $p = 4$ **Or** $\alpha, \alpha + 3 =$ roots $\alpha + \alpha + 3 = -5$ $2\alpha = -8$ $\alpha = -4$ and $\alpha + 3 = -1$ $p = (-1)(-4) = 4$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Demonstrates understanding of 3 as difference of roots e.g. α with $\alpha \pm 3$ $x^2 - x(\text{sum}) + \text{product} = 0$ One correct value for p $x^2 + 5x + p > 0$ Sketch of U-shaped quadratic with turning point above the x-axis *High Partial Credit:* Relevant equation in α (with α as only unknown *Full Credit* (– 1): $p > 6{\cdot}25$
(a) (iii)	$b^2 - 4ac < 0$ $5^2 - 4(1)(p) < 0$ $25 - 4p < 0$ $4p > 25$ $p > 6{\cdot}25$ $p = 7$ and $p = 8$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $b^2 - 4ac$ One correct value for p $x^2 + 5x + p > 0$ *High Partial Credit:* Relevant inequality in p (with p as only unknown *Full credit* (–1): $p > 6{\cdot}25$

<table>
<tr>
<td>(b)</td>
<td>
$-1 \le 2x + 5 \le 1$
$-6 \le 2x \le -4$
$-3 \le x \le -2$

$2x + 5 \le 1$
$2x \le -4$
$x \le -2$

$-1 \le 2x + 5$
$-6 \le 2x$
$-3 \le x$
$-3 \le x \le -2$

Or
$(2x + 5)^2 \le 1$
$4x^2 + 20x + 25 \le 1$
$4x^2 + 20x + 24 \le 0$
$x^2 + 5x + 6 \le 0$
$(x + 2)(x + 3) \le 0$
$x = -2, x = -3$
$-3 \le x \le -2$
</td>
<td>
Scale 5D (0, 2, 3, 4, 5)
Low Partial Credit:
$(2x + 5)^2 \le 1$
one linear inequality

Mid Partial Credit:
$-1 \le 2x + 5 \le 1$
Identifies both linear inequalities
Quadratic inequality involving 0

High Partial Credit:
Finding −3 and −2 in Methods 1 or 2
Roots of quadratic found
$-6 \le 2x \le -4$ or equivalent

Note: Accept $-3 < x < -2$
</td>
</tr>
</table>

<table>
<tr><th>Q2</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>

$$iz_1 = -4 + 3i$$
$$i(iz_1) = i(-4 + 3i)$$
$$-z_1 = -4i + 3i^2$$
$$z_1 = 3 + 4i$$
$$3z_1 - z_2 = 3(3 + 4i) - z_2 = 11 + 17i$$
$$z_2 = 9 + 12i - 11 - 17i$$
$$z_2 = -2 - 5i$$

Or

$$z_1 = \frac{(-4 + 3i)(-i)}{(i)(-1)}$$

$z_1 = 3 + 4i$ and continues</td><td>

Scale 5D (0, 2, 3, 4, 5)
Low Partial Credit:
Either equation multiplied/divided by $\boldsymbol{i}$

Mid Partial Credit:
z_1 found
z_2 written in terms of z_1 with z_1 substituted
z_1 eliminated

High Partial Credit
z_1 found and substituted into second equation
z_2 found by elimination</td></tr>
<tr><td>(b)
(i)</td><td>

$$r = \frac{T_2}{T_1} = \frac{5 - i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$$
$$r = \frac{15 - 13i - 2}{9 + 4}$$
$$r = \frac{13 - 13i}{13}$$
$$r = 1 - i$$</td><td>

Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$\frac{T_2}{T_1}$

High Partial Credit:
$\frac{5 - i}{3 + 2i} \times \frac{3 - 2i}{3 - 2i}$</td></tr>
<tr><td>(b)
(ii)</td><td>

$$T_9 = ar^8$$
$$T_9 = (3 + 2i)(1 - i)^8$$
$$T_9 = (3 + 2i)\left(\sqrt{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)\right)^8$$
$$T_9 = (3 + 2i)\left(\sqrt{2}\right)^8\left(\cos\frac{7\pi(8)}{4} + i\sin\frac{7\pi(8)}{4}\right)$$
$$T_9 = (3 + 2i)(16)(\cos 14\pi + i\sin 14\pi)$$
$$T_9 = (3 + 2i)(16)(1 + 0i)$$
$$T_9 = 48 + 32i$$</td><td>

Scale 15D (0, 4, 7, 11, 15)
Low Partial Credit:
$T_9 = ar^8$
Any correct use of De Moivre
Some use of De Moivre's Theorem on r

Mid Partial Credit:
Modulus and argument found for r

High Partial Credit:
Solution in polar form with some simplification

Note: Accept candidates r from (b)(i)</td></tr>
</table>

Q3	Model Solution – 25 Marks	Marking Notes
(a)	$fg(x) = f\left(\frac{x+5}{6}\right)$ $fg(x) = 6\left(\frac{x+5}{6}\right) - 5 = x$ $gf(x) = g(6x-5)$ $gf(x) = \frac{(6x-5)+5}{6} = \frac{6x}{x} = x$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $f\left(\frac{x+5}{6}\right)$ $g(6x-5)$ Particular case verification *High Partial Credit:* One correct composition simplified to x
(b) (i)	$\log_5 y = \log_5 5x^2$ $\log_5 y = \log_5 5 + \log_5 x^2$ $\log_5 y = 1 + 2\log_5 x$ $a = 1$ and $b = 2$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\log_5 5x^2 = \log_5 y$ $\log_5 y = \log_5 5x^2$ *High Partial Credit:* $\log_5 y = \log_5 5 + \log_5 x^2$
(b) (ii)	$\log_5 y = \log_5 5x^2 = 2 + \log_5\left(\frac{126x}{25} - 1\right)$ $\log_5 5x^2 = \log_5\left(\frac{126x}{25} - 1\right) \times 25$ $5x^2 = 126x - 25$ $5x^2 - 126x + 25 = 0$ $(5x-1)(x-25) = 0$ $x = \frac{1}{5}$ **or** $x = 25$ $y = 5x^2 = 5\left(\frac{1}{5}\right)^2 2 = \frac{1}{5}$ **or** $y = 5(25)^2 = 3125$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Some relevant use of laws of logs *Mid Partial Credit:* Quadratic equation *High Partial Credit:* x values found **Note**: If 2 is incorrectly (non log) dealt with then award MPC at most **Note**: If incorrect work leads to a non-quadratic equation then award MPC at most

Q4	Model Solution – 25 Marks	Marking Notes
(a)	$f'(x) = 3x^2 + 2kx + 15$ $3(3)^2 + 2k(3) + 15 = -12$ $27 + 6k + 15 = -12$ $6k = -54$ $k = -9$	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Any relevant differentiation *Mid Partial Credit:* Expression fully differentiated *High Partial Credit:* Derivative fully substituted *No Credit:* No differentiation
(b)	$f'(x) = 3x^2 + 2(-9)x + 15$ $3x^2 - 18x + 15 = 0$ $x^2 - 6x + 5 = 0$ $x = 1 \quad x = 5$ $f(1) = 15 \qquad (1, 15)$ $f(5) = -17 \qquad (5, -17)$ $m_{g(x)} = -\frac{32}{4} = -8$ $y - 15 = -8(x - 1)$ $g(x)\!: \quad 8x + y - 23 = 0$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Any relevant differentiation *Mid Partial Credit:* Both x values found *High Partial Credit:* Turning points found
(c)	$f''(x) = 6x - 18 = 0$ $x = 3$ $f(3) = -1$ $(3, -1)$ is the point of inflection $8(3) + (-1) - 23 = 0$ $0 = 0$ $\Rightarrow (3, -1) \in g(x)$.	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $f''(x)$ *High Partial Credit:* x coordinate of point of inflection found Point of inflection found **Note**: Accept candidates $g(x)$ from **(b)** with relevant statement

Q5	Model Solution – 25 Marks	Marking Notes
(a)	$A = \frac{250000(0{\cdot}00287)(1{\cdot}00287)^{300}}{(1{\cdot}00287)^{300} - 1}$ $A = €1244{\cdot}06$ **Or** $\frac{A}{1{\cdot}00287^1} + \frac{A}{1{\cdot}00287^2} + \cdots \frac{A}{1{\cdot}00287^{300}}$ $= 250000$ $A\left[\frac{\frac{1}{1{\cdot}00287}\left(\frac{1}{1{\cdot}00287}^{300} - 1\right)}{\frac{1}{1{\cdot}00287} - 1}\right] = 250000$ $200{\cdot}9544372 \times A = 250000$ $A = €1244{\cdot}06$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Formula with some correct substitution $(1{\cdot}00287)$ 300 *High Partial Credit:* Formula fully substituted
(b)	$\frac{1771}{1{\cdot}003} + \frac{1771}{1{\cdot}003^2} + \cdots + \frac{1771}{1{\cdot}003^{167}} + \frac{1771}{1{\cdot}003^{168}}$ $S_{168} = \frac{\frac{1771}{1{\cdot}003}\left[\left(\frac{1}{1{\cdot}003}\right)^{168} - 1\right]}{\frac{1}{1{\cdot}003} - 1}$ $= €233438{\cdot}25$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $\frac{1771}{1{\cdot}003}$ 168 *Mid Partial Credit:* S_{168} formula with some substitution *High Partial Credit:* Formula fully substituted

Q6	Model Solution – 25 Marks	Marking Notes
(a)	$f(x) = (3x-5)(2x+4)$ $= 6x^2 + 2x - 20$ $f(x+h) = 6(x+h)^2 + 2(x+h) - 20$ $= 6x^2 + 12hx + 6h^2 + 2x + 2h - 20$ $f(x+h) - f(x) = 12hx + 6h^2 + 2h$ $\dfrac{f(x+h)-f(x)}{h} = 12x + 6h + 2$ $\lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = 12x + 2$ $f'(x) = 12x + 2$	**Scale 15C (0, 5, 10, 15)** *Low Partial Credit:* Some substitution into $f(x+h)$ or $y + \Delta y$ *High Partial Credit:* $f(x+h) - f(x) = 12hx + 6h^2 + 2h$ *No Credit*: Not from first principles $(3x-5)(2x+4) = 6x^2 + 2x - 20$
(b) **(i)**	$h'(x) = \dfrac{1}{2}\left(\dfrac{1}{2x+3}\right)(2)$ $= \dfrac{1}{2x+3}$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Any relevant differentiation *High Partial Credit:* $\dfrac{1}{2}\left(\dfrac{1}{2x+3}\right)$
(b) **(ii)**	$\displaystyle\int_0^A \frac{1}{2x+3}\,dx = ln3$ $\frac{1}{2}\ln(2x+3)\ \Big\vert_0^A = \ln 3$ $\dfrac{1}{2}(\ln(2A+3) - \ln 3) = \ln 3$ $\dfrac{1}{2}\ln\left(\dfrac{2A+3}{3}\right) = \ln 3$ $\ln\left(\dfrac{2A+3}{3}\right)^{\frac{1}{2}} = \ln 3$ $\left(\dfrac{2A+3}{3}\right)^{\frac{1}{2}} = 3$ $\dfrac{2A+3}{3} = 9$ $2A + 3 = 27$ $2A = 24$ $A = 12$	**Scale 5D (0, 2, 3, 4, 5)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* $\frac{1}{2}\ln(2x+3)\ \Big\vert_0^A$ Substitutes limits into integral and stops Correct integration with some substitution *High Partial Credit:* Integral evaluated at $x = A$ only (i.e. omits $\ln 3$ on LHS and finishes **Note**: Must have integration to gain any credit

<table>
<tr><th>Q7</th><th>Model Solution – 50 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)
(i)</td><td><table><tr><td>T.</td><td>T_1</td><td>T_2</td><td>T_3</td><td>T_4</td><td>T_5</td><td>T_6</td><td>T_7</td><td>T_8</td></tr><tr><td>No.</td><td>1</td><td>3</td><td>6</td><td>10</td><td>15</td><td>21</td><td>28</td><td>36</td></tr></table></td><td>Scale 10C (0, 4, 8, 10)
Low Partial Credit:
One correct new entry

High Partial Credit:
Three correct new entries</td></tr>
<tr><td>(a)
(ii)</td><td>$\frac{n}{2}(n+1) = 1275$
$n^2 + n - 2250 = 0$
$(n-50)(n+51)$
$n = 50$
1275 is the 50th triangular number</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$\frac{n}{2}(n+1) = 1275$

High Partial Credit:
$n = 50$

Note: accept T_{50} as valid reason</td></tr>
<tr><td>(b)
(i)</td><td>$T_{n+1} = T_n + (n+1)$
$= \frac{n}{2}(n+1) + (n+1)$
$= \frac{n(n+1) + 2(n+1)}{2}$
$= \frac{(n+1)(n+2)}{2}$</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
2 identified as C.D.
Correct numerator

High Partial Credit:
$= \frac{n(n+1) + 2(n+1)}{2}$</td></tr>
<tr><td>(b)
(ii)</td><td>$T_{n+1} + T_n$
$= \frac{(n+1)(n+2)}{2} + \frac{n}{2}(n+1)$
$= \frac{(n+1)(2n+2)}{2}$
$= \frac{2(n+1)(n+1)}{2}$
$= (n+1)^2$</td><td>Scale 5C (0, 3, 4, 5)
Low Partial Credit:
$T_{n+1} + T_n$ with some substitution
Particular case verification

High Partial Credit:
$\frac{(n+1)(n+2)}{2} + \frac{n}{2}(n+1)$</td></tr>
</table>

(b) (iii)	$(n+1)^2 = 12544$ $n+1 = \sqrt{12544} = 112$ $n = 111$ $n = 111$ T_{111} is the smaller term $T_{111} = \frac{111(112)}{2}$ $T_{111} = 6216$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $(n+1)^2$ *High Partial Credit:* $n = 111$, or $n = 112$
(c)	$N_3 = \left(\frac{(3+2\sqrt{2})^3 - (3-2\sqrt{2})^3}{4\sqrt{2}}\right)^2$ $= 1225$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* Formula with some substitution *High Partial Credit:* Formula fully substituted *Full Credit:* Correct answer with no work shown

(d)		
	$1^2 + 2^2 + 3^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$ $\boldsymbol{P(1)}$: $1 = \frac{1(2)(3)}{6}$ $\boldsymbol{P(k)}$: $1 + 4 + 9 + \cdots + k^2 = \dfrac{k(k+1)(2k+1)}{6}$ $\boldsymbol{P(k+1)}$: $1 + 4 + 9 + \cdots + k^2 + (k+1)^2 = \dfrac{(k+1)(k+2)(2k+3)}{6}$ $LHS = \dfrac{k(k+1)(2k+1)}{6} + (k+1)^2$ $LHS = \dfrac{k(k+1)(2k+1) + 6(k+1)^2}{6}$ $LHS = \dfrac{(k+1)[k(2k+1) + 6(k+1)]}{6}$ $LHS = \dfrac{(k+1)[2k^2 + 7k + 6]}{6}$ $\dfrac{(k+1)(k+2)(2k+3)}{6} = RHS$ Thus the proposition is true for $n = k+1$ provided it is true for $n = k$ but it is true for $n = 1$ and therefore true for all positive integers.	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* Step $P(1)$ *Mid Partial Credit:* Step $P(k+1)$ *High Partial Credit:* Uses *Step* $P(k)$ to prove *Step* $P(k+1)$ *Full Credit(–1):* Concluding statement missing **Note:** Accept *Step* $P(1)$, *Step* $P(k)$, *Step* $P(k+1)$ in any order

Q8	Model Solution – 45 Marks	Marking Notes
(a) (i)	$\cos\theta = \frac{x}{5}$ $\sin\theta = \frac{y}{5}$ $5\cos\theta = x$ $5\sin\theta = y$ $(x, y) = (5\cos\theta\,,\ 5\sin\theta)$ $\therefore a = 5,\ \ b = 5$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $\cos\theta = \frac{x}{5}$ or equivalent *High Partial Credit:* a or b found Correct answer without work
(a) (ii)	$A(\theta) = (10\cos\theta) \times (10\sin\theta)$ $A(\theta) = 100\cos\theta\sin\theta$ $= 50 \times 2\cos\theta\sin\theta$ $= 50(\sin 2\theta)$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* xy $(10\cos\theta) \times (10\sin\theta)$ *High Partial Credit:* $100\cos\theta\sin\theta$
(a) (iii)	$A(\theta) = 50\sin 2\theta$ $A'(\theta) = 50\cos 2\theta \times 2$ $A'(\theta) = 100\cos 2\theta = 0$ $\cos 2\theta = 0$ $2\theta = \frac{\pi}{2}$ $\theta = \frac{\pi}{4}$ $2x = 2\left(5\cos\left(\frac{\pi}{4}\right)\right) = 5\sqrt{2}$ $2y = 2\left(5\sin\left(\frac{\pi}{4}\right)\right) = 5\sqrt{2}$ $\Longrightarrow$ Square	**Scale 15D (0, 4, 7, 11, 15)** *Low Partial Credit:* $a'(\theta)$ States $\frac{dy}{dx} = 0$ *Mid Partial Credit:* Correct differentiation *High Partial Credit:* Value of θ at maximum found Value of x or y at maximum fully substituted *No Credit:* No differentiation
(a) (iv)	Max area $= 5\sqrt{2} \times 5\sqrt{2}$ $=$ 50 Square units **Or** Max area $= 50(\sin 2\theta)$ $50(\sin\frac{\pi}{2})$ $=$ 50 Square units	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* xy length $\times$ width $50(\sin 2\theta)$ *High Partial Credit:* Area formula fully substituted

(b)	$$\frac{dx}{dt}=\frac{dx}{dl}\cdot\frac{dl}{dt}$$ $$\frac{2}{5}=\frac{x}{l+x}$$ $$2l+2x=5x$$ $$x=\frac{2}{3}l$$ $$\frac{dx}{dl}=\frac{2}{3}$$ $$\frac{dx}{dt}=\frac{2}{3}\times\frac{3}{2}$$ $$\frac{dx}{dt}=1\text{ m/sec}$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* $\frac{dx}{dt}$ or $\frac{dx}{dl}$ or $\frac{dl}{dt}$ given Reference to similar triangles $\frac{2}{5}$ or $\frac{5}{2}$ *Mid Partial Credit:* $\frac{dx}{dt}=\frac{dx}{dl}\cdot\frac{dl}{dt}$ or equivalent with one relevant substitution $x=\frac{2}{3}l$ *High Partial Credit:* $\frac{dx}{dl}$ and $\frac{dl}{dt}$ found

Q9	Model Solution – 55 Marks	Marking Notes
(a) **(i)**	$N(t) = 450e^{0\cdot065t}$ $N(4\cdot5) = 450e^{0\cdot065(4\cdot5)}$ $= 602\cdot89$ $= 603$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some substitution into function Correct answer without work *High Partial Credit:* $450e^{0\cdot065(4\cdot5)}$
(a) **(ii)**	$N(t) = 450e^{0\cdot065t}$ $\frac{N(t)}{450} = e^{0\cdot065t}$ Convert to log equation $\ln(\frac{N(t)}{450}) = 0\cdot065t$ $\ln(N(t)) - \ln 450 = 0\cdot065t$ $\frac{\ln(N(t)) - \ln 450}{0\cdot065} = t$ $t = \frac{\ln(790) - \ln 450}{0\cdot065}$ $t = 8\cdot7$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Some substitution into function Full substitution and stops *High Partial Credit:* Equation in t (i.e. logs handled correctly)

(b)	$\frac{1}{9}\int_3^{12} 450e^{0\cdot065t}dt$ $= \frac{450}{9(0\cdot065)}\left[e^{12(\cdot065)} - e^{3(0\cdot065)}\right]$ $= 743\cdot2$ Average no. $= 743$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Integration indicated *Mid Partial Credit:* Integration correct *High Partial Credit:* Substitutes limits into integral and stops **Note**: Must have integration to gain any credit
(c)	$N'(t) = 450e^{0\cdot065t} \times 0\cdot065$ $N'(t) = 29\cdot25e^{0.065t}$ $N'(12) = 29\cdot25e^{0\cdot065(12)}$ $= 63\cdot8$ At hour 12 the population is growing at a rate of 64 bacteria per hour **or** At hour 12 the population is growing at a rate of 63·8 bacteria per hour	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $N'(t)$ stated or indicated *High Partial Credit:* Derivative fully substituted $N'(12) = 63\cdot8$ and stops

(d)	$N'(t) = 29{\cdot}25e^{0{\cdot}065k} > 90$ $e^{0{\cdot}065k} > 29{\cdot}25$ $k > \dfrac{\ln \frac{90}{29{\cdot}25}}{0{\cdot}065}$ $k > 17{\cdot}29$ $k = 18$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $29{\cdot}25e^{0{\cdot}065k} > 90$ *High Partial Credit:* Equation in k (i.e. taking logs handled correctly) *No Credit:* No differentiation **Note:** if $k > 17{\cdot}29 \Rightarrow k = 17$ Award Full credit (–1)
(e)	$450e^{0{\cdot}065t} = 220e^{0{\cdot}17t}$ $\dfrac{450}{220} = \dfrac{e^{0{\cdot}17t}}{e^{0{\cdot}065t}}$ $\dfrac{450}{220} = e^{0{\cdot}105t}$ $\ln\left(\dfrac{450}{220}\right) = 0{\cdot}105t$ $\dfrac{\ln\left(\frac{450}{220}\right)}{0{\cdot}105} = t$ $t = 6{\cdot}82$ t = 7 hours	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* $450e^{0{\cdot}065t} = 220e^{0{\cdot}17t}$ *High Partial Credit:* Equation in t (i.e. taking logs handled correctly)