Coimisiún na Scrúduithe Stáit
State Examinations Commission

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Matamaitic	**Ardleibhéal**

Marking Scheme	Leaving Certificate Examination, 2007
Maths	**Higher Level**

Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE MATHS

HIGHER LEVEL

MARKING SCHEME

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MARKING SCHEME
LEAVING CERTIFICATE EXAMINATION 2007

MATHEMATICS – HIGHER LEVEL – PAPER 1

GENERAL GUIDELINES FOR EXAMINERS – PAPER 1

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work of merit is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**

Part (a) **10 (5, 5)marks** **Att (2, 2)**

1. **(a)** Simplify $\dfrac{x^2 - xy}{x^2 - y^2}$.

Factors: **5 marks** **Att 2**
Cancellation: **5 marks** **Att 2**

1 (a)

$$\frac{x^2 - xy}{x^2 - y^2} = \frac{x(x-y)}{(x+y)(x-y)} = \frac{x}{x+y}$$

Blunders (-3)

B1 Factors once only
B2 Indices
B3 Incorrect cancellation

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

1 (b) Let $f(x) = x^2 + (k+1)x - k - 2$, where k is a constant.

(i) Find the value of k for which $f(x) = 0$ has equal roots.

(ii) Find, in terms of k, the roots of $f(x) = 0$.

(iii) Find the range of values of k for which both roots are positive.

(i) $\boldsymbol{b^2 - 4ac = 0}$ **applied:**	**5 marks**	**Att 2**
Finish:	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**
(iii)	**5 marks**	**Att 2**

1 (b)

(i) $f(x) = 0 \Rightarrow x^2 + (k+1)x + (-k-2) = 0$

Equal roots: $b^2 - 4ac = 0$

$$(k+1)^2 - 4(1)(-k-2) = 0$$
$$k^2 + 2k + 1 + 4k + 8 = 0$$
$$k^2 + 6k + 9 = 0$$
$$(k+3)^2 = 0 \qquad \Rightarrow k = -3$$

1 b(ii)

Roots of $f(x)=0$

$$x=\frac{-(k+1)\pm\sqrt{(k+1)^2-4(1)(-k-2)}}{2}$$

$$=\frac{-(k+1)\pm\sqrt{k^2+6k+9}}{2}$$

$$=\frac{-(k+1)\pm(k+3)}{2}$$

$x=\frac{-k-1+k+3}{2}$ **or** $x=\frac{-k-1-k-3}{2}$

$x=\frac{2}{2}$ **or** $x=\frac{-2k-4}{2}$

$x=1$ **or** $x=-k-2$

or

1 b(ii)

$f(x)=(x-1)(x+k+2)=0$

$x-1=0$ or $x+k+2=0$

$x=1$ or $x=-k-2$

1b(iii)

Positive roots : $(-k-2)>0$

$-2>k$

Blunders (-3)

B1 Equal roots condition

B2 Expansion of $(k+1)^2$ once only

B3 Indices

B4 Factors

B5 Roots formula once only

B6 Inequality sign

B7 Deduction of root from factor or no root

B8 Range

Slips (-1)

S1 Numerical

Attempts

A1 Equation not quadratic in b(i) gives $A\neq 2$ at most in finish

A2 Using remainder theorem

Part (c)	20 (5, 5, 10) marks	Att (2, 2, 3)

1 (c) $x+p$ is a factor of both ax^2+b and $ax^2+bx-ac$.

(i) Show that $p^2=-\frac{b}{a}$ and that $p=\frac{-b-ac}{b}$.

(ii) Hence show that $p^2+p^3=c$.

(i) Show p^2	**5 marks**	**Att 2**
Show p	**5 marks**	**Att 2**
(ii)	**10 marks**	**Att 3**

1 (c) (i)

$$(x+p)\text{ factor of } ax^2+b \Rightarrow f(-p)=0$$
$$a(-p)^2+b=0$$
$$ap^2=-b$$
$$p^2=\frac{-b}{a}$$

$$(x+p)\text{ factor of } ax^2+bx-ac \Rightarrow f(-p)=0$$
$$a(-p)^2+b(-p)-ac=0$$
$$ap^2-bp-ac=0$$
But $ap^2=-b$ from above $\Rightarrow -b-ac=bp$

$$\frac{-b-ac}{b}=p$$

or

1 (c) (i)

$$\begin{array}{r} ax-ap \\ x+p\,\big)\,\overline{ax^2+b} \\ \underline{ax^2+apx} \\ -apx+b \\ \underline{-apx-ap^2} \\ ap^2+b \end{array}$$

Since $(x-p)$ factor $\Rightarrow ap^2+b=0$
$$ap^2=-b$$
$$p^2=\frac{-b}{a}$$

$$\begin{array}{r} ax+(b-ap) \\ x+p\,\big)\,\overline{ax^2+bx-ac} \\ \underline{ax^2+apx} \\ (b-ap)x-ac \\ \underline{(b-ap)x+p(b-ap)} \\ -p(b-ap)-ac \end{array}$$

Since $(x+p)$ factor
$$-pb+ap^2-ac=0$$
$$-pb-b-ac=0$$
$$-b-ac=pb$$
$$\frac{-b-ac}{b}=p$$

1(c)(ii)

$$
\begin{aligned}
p^2 + p^3 &= p^2(1+p) \\
&= \frac{-b}{a}\left(1 + \frac{-b-ac}{b}\right) \\
&= \frac{-b}{a}\left(\frac{b-b-ac}{b}\right) \\
&= \frac{bac}{ab} \\
&= c
\end{aligned}
$$

Blunders (-3)

B1 Deduction root from factor
B2 Indices
B3 Not in required form

Slips (-1)

S1 Not changing sign when subtracting in division.

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**15 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

2 (a) Without using a calculator, solve the simultaneous equations

$$x + y + z = 2$$
$$2x + y + z = 3$$
$$x - 2y + 2z = 15.$$

(a) **10 marks** **Att 3**

2 (a)

(i) $x + y + z = 2$

(ii) $2x + y + z = 3$

(iii) $x - 2y + 2z = 15$

(ii) $2x + y + z = 3$

(i) $x + y + z = 2$

$x = 1$

(ii) $2x + y + z = 3$

$y + z = 1$

(iii) $1 - 2y + 2z = 15$

$-2y + 2z = 14$

$-y + z = 7$

(ii) $y + z = 1$

(iii) $-y + z = 7$

$2z = 8$

$z = 4$

(ii) $y + z = 1$

$y + 4 = 1$

$y = -3$

$x = 1 \qquad y = -3 \qquad z = 4$

Blunders (-3)

B1 Multiplying one side of equation only

B2 Not finding 2nd unknown (having found 1st)

B3 Not finding 3rd unknown (having found 1st and 2nd)

Slips (-1)

S1 Numerical

S2 Not changing sign when subtracting

Worthless

W1 Trial and error only

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

2 (b) α and β are the roots of the equation $x^2 - 4x + 6 = 0$.

(i) Find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

(ii) Find the quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

Values of $(\alpha+\beta)$ & $\alpha\beta$ or solve quadr.	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**
Correct Statement	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

2 (b) (i)

$$x^2 - (4)x + (6) = 0$$

$$x^2 - (\alpha + \beta)x + (\alpha\beta) = 0$$

$$\therefore \quad \alpha + \beta = 4 \qquad \alpha\beta = 6$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{4}{6} = \frac{2}{3}$$

2 (b) (ii)

$$x^2 - (\text{sum of roots})\, x + (\text{product roots}) = 0$$

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha} \cdot \frac{1}{\beta}\right) = 0$$

$$x^2 - \left(\frac{2}{3}\right)x + \left(\frac{1}{6}\right) = 0$$

$$6x^2 - 4x + 1 = 0$$

Blunders (-3)

B1 Indices
B2 Incorrect sum
B3 Incorrect product
B4 Statement incorrect

Slips (-1)

S1 Numerical
S2 Not as equation

Attempts

A1 Not quadratic equation

Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

2 (c) (i) Prove that $x+\dfrac{9}{x+2} \geq 4$, where $x+2>0$.

(ii) Prove that $x+\dfrac{9}{x+a} \geq 6-a$, where $x+a>0$.

(i) Quadratic Inequality	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**
(ii) Quadratic Inequality	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

2 (c) (i)

$$x+\frac{9}{x+2}\geq 4$$

$$\Leftrightarrow x(x+2)+9\geq 4(x+2), \quad [\text{given } x+2>0]$$

$$\Leftrightarrow x^2+2x+9\geq 4x+8$$

$$\Leftrightarrow x^2-2x+1\geq 0$$

$$\Leftrightarrow (x-1)^2\geq 0 \quad \text{True}$$

or

$$x+\frac{9}{x+2}-4$$

$$=\frac{x(x+2)+9-4(x+2)}{(x+2)}$$

$$=\frac{x^2+2x+9-4x-8}{x+2}$$

$$=\frac{x^2-2x+1}{x+2}$$

$$=\frac{(x-1)^2}{(x+2)}\geq 0, \text{ which is true,}$$

[given $x+2>0$]

2 (c) (ii)

$$x+\frac{9}{x+a}\geq 6-a$$

$$\Leftrightarrow x(x+a)+9\geq (6-a)(x+a)$$

[given $(x+a)>0$]

$$\Leftrightarrow x^2+ax+9\geq 6x-ax+6a-a^2$$

$$\Leftrightarrow x^2+2ax-6x+a^2-6a+9\geq 0$$

$$\Leftrightarrow x^2+2(a-3)x+(a-3)^2\geq 0$$

Let $y=a-3$

$$\Leftrightarrow x^2+2yx+y^2\geq 0$$

$$\Leftrightarrow (x+y)^2\geq 0$$

$$\Leftrightarrow (x+a-3)^2\geq 0$$

which is true.

or

$$\left(x+\frac{9}{x-a}\right)-(6-a)$$

$$=\frac{x(x+a)+9-(6-a)(x+a)}{x+a}$$

$$=\frac{x^2+ax+9-(6x-ax+6a-a^2)}{x+a}$$

$$=\frac{x^2+2ax-6x+a^2-6a+9}{(x+a)}$$

$$=\frac{x^2+2(a-3)x+(a-3)^2}{(x+a)}$$

Let $y=a-3$

$$=\frac{x^2+2yx+y^2}{x+a}$$

$$=\frac{(x+y)^2}{x+a}$$

$$=\frac{(x+a-3)^2}{(x+a)}$$

≥ 0, given $(x+a)>0$.

Blunders (-3)

B1 Inequality sign
B2 Factors
B3 Incorrect deduction or no deduction

Attempts

A1 Multiplication by $(x+2)^2$
A2 Multiplication by $(x+a)^2$

Worthless

W1 Squares both sides

QUESTION 3

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att 2**

3

(a) Let $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & \frac{3}{2} \end{pmatrix}$. Find $A^2 - 2A.$.

A^2	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

3 (a)

$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & \frac{3}{2} \end{pmatrix}$$

$$A^2 = A.A = \begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & \frac{3}{2} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ 6 & 3 \end{pmatrix}$$

$$2A = 2\begin{pmatrix} \frac{1}{2} & \frac{1}{4} \\ 3 & \frac{3}{2} \end{pmatrix} = \begin{pmatrix} 1 & \frac{1}{2} \\ 6 & 3 \end{pmatrix}$$

$$A^2 - 2A = \begin{pmatrix} 1 & \frac{1}{2} \\ 6 & 3 \end{pmatrix} - \begin{pmatrix} 1 & \frac{1}{2} \\ 6 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Slips

S1 Incorrect element

S2 Numerical

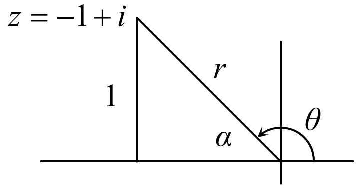
Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

3 (b)	Let $z = -1 + i$, where $i^2 = -1$.	
	(i)	Use De Moivre's theorem to evaluate z^5 and z^9.
	(ii)	Show that $z^5 + z^9 = 12z$.

(i) z in Polar Form	**5 marks**	**Att 2**
z^5	**5 marks**	**Att 2**
z^9	**5 marks**	**Att 2**
(ii) Show	**5 marks**	**Att 2**

3 (b) (i)

$$z = -1 + i$$
$$= r[\cos\theta + i\sin\theta]$$
$$z = \sqrt{2}\left[\cos\tfrac{3\pi}{4} + i\sin\tfrac{3\pi}{4}\right]$$



$$r = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$\tan\alpha = \tfrac{1}{1} = 1$$
$$\therefore \alpha = \tfrac{\pi}{4} \quad \theta = \tfrac{3\pi}{4}$$

(i)

$$z^5 = \left[\sqrt{2}\left(\cos\tfrac{3\pi}{4} + i\sin\tfrac{3\pi}{4}\right)\right]^5$$
$$= 2^{\frac{5}{2}}\left(\cos\tfrac{15\pi}{4} + i\sin\tfrac{15\pi}{4}\right)$$
$$= 2^{\frac{5}{2}}\left(\cos\tfrac{7\pi}{4} + i\sin\tfrac{7\pi}{4}\right)$$
$$= 2^{\frac{5}{2}}\left[\tfrac{1}{\sqrt{2}} - \tfrac{i}{\sqrt{2}}\right]$$
$$= 2^2(1 - i) = 4 - 4i$$

$$z^9 = \left[2^{\frac{1}{2}}\left(\cos\tfrac{3\pi}{4} + i\sin\tfrac{3\pi}{4}\right)\right]^9$$
$$= 2^{\frac{9}{2}}\left[\cos\tfrac{27\pi}{4} + i\sin\tfrac{27\pi}{4}\right]$$
$$= 2^{\frac{9}{2}}\left[\cos\tfrac{3\pi}{4} + i\sin\tfrac{3\pi}{4}\right]$$
$$= 2^{\frac{9}{2}}\left[-\tfrac{1}{\sqrt{2}} + \tfrac{i}{\sqrt{2}}\right]$$
$$= 2^4(-1 + i) = -16 + 16i.$$

3b(ii)

$$z^5 + z^9 = (4 - 4i) + (-16 + 16i)$$
$$= 4 - 4i - 16 + 16i$$
$$= -12 + 12i$$
$$= 12(-1 + i)$$
$$= 12z$$

Blunders (-3)

B1 Argument
B2 Modulus
B3 Trig definition
B4 Indices
B5 *i*
B6 Statement De Moivre once only
B7 Application De Moivre

Slips (-1)

S1 Trig value

Part (c)	**20 (5, 5, 5, 5)**	**Att (2, 2, 2, 2)**

3(c) (i) Find the two complex numbers $a+bi$ for which $(a+bi)^2 = 15+8i$.

(ii) Solve the equation $iz^2 + (2-3i)z + (-5+5i) = 0$.

(i) Quadratic Equation	**5 marks**	**Att 2**
Complex numbers	**5 marks**	**Att 2**

3(c)(i)

$$(a+bi)^2 = 15i + 8i$$
$$a^2 + 2abi + b^2i^2 = (15) + (8)i$$
$$(a^2 - b^2) + (2ab)i = (15) + (8)i$$

(i): $a^2 - b^2 = 15$

(ii): $2ab = 8$

(ii): $2ab = 8 \quad \Rightarrow \quad ab = 4 \quad \Rightarrow \quad b = \frac{4}{a}$

Substitute into (i): $\Rightarrow$ $a^2 - \left(\frac{4}{a}\right)^2 = 15$

$$a^2 - \frac{16}{a^2} = 15$$

Let $y = a^2$, (so $y > 0$)

$$\therefore \quad y - \frac{16}{y} = 15$$
$$y^2 - 16 = 15y$$
$$y^2 - 15y - 16 = 0$$
$$(y-16)(y+1) = 0$$
$$y - 16 = 0 \quad \text{or} \quad y + 1 = 0$$
$$y = 16 \quad \text{or} \quad y = -1$$
$$y = a^2 \neq -1$$
$$\therefore \quad a^2 = 16$$
$$a = \pm 4$$

$a = 4: \ b = \frac{4}{4} = 1 \Rightarrow 4 + i = z_1$

$a = -4: \ b = \frac{4}{-4} = -1 \Rightarrow -4 - i = z_2$

Blunders (-3)

B1 Expansion $(a+ib)^2$
B2 Indices
B3 i
B4 Not like to like
B5 Factors
B6 Quadratic formula
B7 Excess values (not real)
B8 Only one complex number found
B9 Incorrect deduction of root from factor

(ii) Using square root value **5 marks** **Att 2**
Completion **5 marks** **Att 2**

3 (c) (ii)

$$iz^2 + (2-3i)z + (-5+5i) = 0$$

$$a = i \quad ; \quad b = (2-3i) \quad ; \quad c = (-5+5i)$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(2-3i) \pm \sqrt{(2-3i)^2 - 4(i)(-5+5i)}}{2i}$$

$$= \frac{-2+3i \pm \sqrt{4-12i+9i^2+20i-20i^2}}{2i}$$

$$= \frac{-2+3i \pm \sqrt{15+8i}}{2i}$$

$$= \frac{-2+3i \pm (4+i)}{2i}$$

$$z_1 = \frac{-2+3i+(4+i)}{2i} = \frac{2+4i}{i} = \frac{1+2i}{i}\cdot\frac{i}{i} = \frac{i-2}{-1}$$

$$z_1 = 2-i$$

$$z_2 = \frac{-2+3i-(4+i)}{2i} = \frac{-6+2i}{2i} = \frac{-3+i}{i}\cdot\frac{i}{i} = \frac{-3i-1}{-1}$$

$$z_2 = 1+3i$$

Blunders (-3)

B1 Indices
B2 i
B3 Expansion of $(2-3i)^2$ once only
B4 Root formula once only
B5 i in denominator

Slips (-1)

S1 Numerical

QUESTION 4

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

4 (a) Show that $\binom{n}{1} + \binom{n}{2} = \binom{n+1}{2}$ for all natural numbers $n \geq 2$.

(a) L.H.S.	**5 marks**	**Att 2**
R.H.S.	**5 marks**	**Att 2**

4 (a)

$$\text{L.H.S.} = \binom{n}{1} + \binom{n}{2} = n + \frac{n(n\text{-}1)}{2}$$

$$= \frac{2n + n^2 - n}{2}$$

$$= \frac{n^2 + n}{2}$$

$$\text{R.H.S.} = \binom{n+1}{2} = \frac{(n+1)(n)}{2} = \frac{n^2 + n}{2}$$

$$\therefore \binom{n}{1} + \binom{n}{2} = \binom{n+1}{2}$$

Blunders (-3)

B1 Indices

B2 Value $\binom{n}{r}$

Attempts

A1 Correct by using values for n .

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

4 (b) $u_1 = 5$ and $u_{n+1} = \frac{n}{n+1} u_n$ for all $n \geq 1, x \in \mathbf{N}$.

(i) Write down the values of u_2, u_3, and u_4.

(ii) Hence, by inspection, write an expression for u_n in terms of n.

(iii) Use induction to justify your answer for part **(ii)**.

(i) values	**5 marks**	**Att 2**
(ii) u_n	**5 marks**	**Att 2**
(iii) P(1) and P(*k*)	**5 marks**	**Att 2**
P(*k*+1)	**5 marks**	**Att 2**

4 (b)

(i)

$u_1 = 5 = \frac{5}{1}$

$u_2 = \frac{1}{1+1}(5) = \frac{5}{2}$

$u_3 = \frac{2}{3}\left(\frac{5}{2}\right) = \frac{5}{3}$

$u_4 = \frac{3}{4}\left(\frac{5}{3}\right) = \frac{5}{4}$

(ii)

$u_n = \frac{5}{n}$

(iii) To prove : $u_n = \frac{5}{n}$

P(1): $n = 1$: $u_1 = \frac{5}{1} \Rightarrow$ true for $n = 1$

Assume P(*k*): i.e., assume true for $n = k \Rightarrow u_k = \frac{5}{k}$.

Deduce P(*k* + 1): i.e., prove true for $n = k + 1$:

$u_{k+1} = \frac{k}{k+1}(u_k) = \frac{k}{k+1}\left(\frac{5}{k}\right) = \frac{5}{k+1}$

$\therefore$ truth of P(*k*) implies truth of P(*k*+1), and P(1) is true.

$\therefore$ true for all $n \geq 1$.

* Note: Accept P(1) as given

Blunders (-3)

B1 Incorrect term once only

B2 Incorrect deduction

B3 Incorrect P(*k*)

B4 Incorrect P(*k* + 1)

Part (c)	20 (5, 5, 5, 5) marks	Att(2, 2, 2, 2)

4 (c) The sum of the first n terms of a series is given by $S_n = n^2 \log_e 3$.

(i) Find the n^{th} term and prove that the series is arithmetic.

(ii) How many of the terms of the series are less than $12 \log_e 27$?

(i) u_n	**5 marks**	**Att 2**
Prove AP	**5 marks**	**Att 2**
(ii) Inequality	**5 marks**	**Att 2**
No of terms	**5 marks**	**Att 2**

4 (c) (i)

$$S_n = n^2 \ln 3$$

$$S_{n-1} = (n-1)^2 \ln 3$$

$$u_n = S_n - S_{n-1} = n^2 \ln 3 - (n-1)^2 \ln 3$$

$$= (\ln 3)\left[n^2 - (n^2 - 2n + 1)\right]$$

$$= (\ln 3)[2n - 1]$$

$$u_n = (2n-1)\ln 3$$

$$u_{n+1} = [2(n+1) - 1]\ln 3 = (2n+1)\ln 3$$

$$d = u_{n+1} - u_n = (2n+1)\ln 3 - (2n-1)\ln 3$$

$$= \ln 3[2n + 1 - 2n + 1]$$

$$= 2\ln 3 \quad \text{constant}$$

$\therefore$ arithmetic, with $d = 2\ln 3$

4(c)(ii) $12\ln 27 = 12\ln(3^3) = 12[3\ln 3] = 36\ln 3$

Let $(2n-1)\ln 3 \le 36\ln 3$

$$2n - 1 \le 36$$

$$2n \le 37$$

$$n \le 18\tfrac{1}{2}$$

So the first 18 terms are less than $12\ln 27$.

Blunders (-3)

B1 AP formula once only
B2 Incorrect terms (must be consecutive)
B3 Log laws
B4 Indices
B5 Incorrect *ln* 27 or no *ln* 27
B6 Inequality sign
B7 Incorrect value or no value
B8 $U_n = S_{n+1} - S_n$

Slips (-1)

S1 Numerical

QUESTION 5

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

5 (a) Plot, on the number line, the values of x that satisfy the inequality

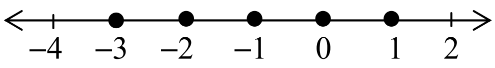
$$|x+1| \le 2, \text{ where } x \in \mathbf{Z}.$$

Inequality	**5 marks**	**Att 2**
Solution set plotted	**5 marks**	**Att 2**

5 (a)

$$|x+1| \le 2 \ \Rightarrow\ -2 \le x+1 \le 2.$$

$$\therefore\ -3 \le x \le 1.$$



or

5(a)

$$|x+1| \le 2$$

$$(x+1)^2 \le (2)^2$$

$$x^2+2x+1 \le 4$$

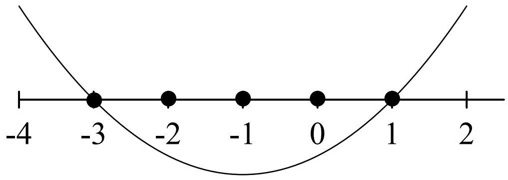
$$x^2+2x-3 \le 0$$

Graph: $y = x^2+2x-3$

roots: $(x+3)(x-1)=0$

$x=-3, x=1$

$\therefore\ -3 \le x \le 1.$



Blunders (-3)

B1 Upper limit
B2 Lower limit
B3 Expansion $(x+1)^2$ once only
B4 Inequality sign
B5 Indices
B6 Factors once only
B7 Root formula once only
B8 Deduction root from factor
B9 Incorrect range
B10 Set not plotted

Slips (-1)

S1 Numerical

Attempts

A1 One inequality sign
A2 Inequality signs ignored
A3 Scaled and numbered line

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

5 (b) In the expansion of $\left(2x - \frac{1}{x^2}\right)^9$,

(i) find the general term.

(ii) find the value of the term independent of x.

General Term	**5 marks**	**Att 2**
$\boldsymbol{u_{r+1}}$	**5 marks**	**Att 2**
Power of $\boldsymbol{x}$	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**

5 (b) $\left[2x + \left(-\frac{1}{x^2}\right)\right]^9$

(i) General Term:

$$u_{n+1} = \binom{9}{r}(2x)^{9-r}\left(-\frac{1}{x^2}\right)^r$$
$$= \binom{9}{r}(2)^{9-r}.x^{9-r}.\left(-x^{-2}\right)^r$$
$$= k \quad x^{9-r}.x^{-2r}$$
$$= k.x^{9-3r}$$

(ii) Term independent or x is the term with x° :

$$9 - 3r = 0 \Rightarrow r = 3$$
$$u_4 = \binom{9}{3}(2x)^6\left(-\frac{1}{x^2}\right)^3 = \frac{9.8.7}{1.2.3}\left(64x^6\right)\left(-\frac{1}{x^6}\right)$$
$$= \text{-}5376$$

or

(ii) $\left[2x + \left(-\frac{1}{x^2}\right)\right]^9$

$$= (2x)^9 + \binom{9}{1}(2x)^8\left(-\frac{1}{x^2}\right)^1 + \binom{9}{2}(2x)^7\left(-\frac{1}{x^2}\right)^2 + \binom{9}{3}(2x)^6\left(-\frac{1}{x^2}\right)^3 + ...$$

$$u_4 \text{ has } x^\circ \Rightarrow u_4 = \binom{9}{3}(2x)^6\left(-\frac{1}{x^2}\right)^3$$
$$= \frac{9.8.7}{1.2.3}(2)^6\left(x^6\right)(-1)^3\frac{1}{x^6}$$
$$= \text{-}5376$$

Blunders (-3)

B1 General term

B2 Error Binomial expansion once only

B3 Indices

B4 Value $\binom{n}{r}$ or no value $\binom{n}{r}$

B5 $x^\circ \neq 1$

B6 Correct term in expansion not identified.

Slips (-1)

S1 Numerical

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

5 (c) The n^{th} term of a series is given by nx^n, where $|x| < 1$.

(i) Find an expression for S_n, the sum to n terms of the series.

(ii) Hence, find the sum to infinity of the series.

(i) xS_n	**5 marks**	**Att 2**
Correct GP evaluated	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**
(ii) Sum to infinity	**5 marks**	**Att 2**

5 (c) (i)

$$s_n = x + 2x^2 + 3x^3 + \dots\dots + (n-1)x^{n-1} + nx^n$$

$$xs_n = \quad x^2 + 2x^3 + \dots\dots + (n-1)x^n + nx^{n+1}$$

$$s_n - xs_n = x + x^2 + x^3 + \dots\dots + x^n - nx^{n+1}$$

$$= \left[x + x^2 + x^3 + \dots\dots + x^n\right] - nx^{n+1}$$

$$= [\text{G.P to } n \text{ terms with } a = x \text{ and } r = x] - nx^{n+1}$$

$$s_n(1-x) = \frac{x(1-x^n)}{1-x} - nx^{n+1}$$

$$s_n = \frac{x(1-x^n)}{(1-x)^2} - \frac{nx^{n+1}}{(1-x)}$$

5(c)(ii)

$|x| < 1$. Hence, as $n \to \infty$, $x^n \to 0$.

$$s_\infty = \frac{x(1-0)}{(1-x)^2} - \frac{0}{1-x}$$

$$s_\infty = \frac{x}{(1-x)^2}$$

Blunders (-3)

B1 Indices

B2 GP formula

B3 Incorrect *'a'*

B4 Incorrect *'r'*

B5 S_n not isolated

B6 $x^n \to 0$ in (ii)

Slips (-1)

S1 Not changing sign when subtracting

QUESTION 6

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

6 (a) Differentiate $\frac{x^2-1}{x^2+1}$ with respect to x.

(a) **10 marks** **Att 3**

6 (a)

$$y=\frac{x^2-1}{x^2+1}$$

$$\frac{dy}{dx}=\frac{(x^2+1)(2x)-(x^2-1)(2x)}{(x^2+1)^2}=\frac{2x^3+2x-2x^3+2x}{(x^2+1)^2}$$

$$=\frac{4x}{(x^2+1)^2}$$

Blunders (-3)
B1 Indices
B2 Differentiation

Attempts
A1 Error in differentiation formula

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

6 (b) (i) Differentiate $\frac{1}{x}$ with respect to x from first principles.

(ii) Find the equation of the tangent to $y=\frac{1}{x}$ at the point $\left(2,\frac{1}{2}\right)$.

(i) $\boldsymbol{f(x+h)-f(x)}$ **simplified** **5 marks** **Att 2**
Finish **5 marks** **Att 2**

6 (b) (i)

$$f(x)=\frac{1}{x} \qquad f(x+h)=\frac{1}{x+h}$$

$$f(x+h)-f(x)=\frac{1}{x+h}-\frac{1}{x}$$

$$=\frac{x-(x+h)}{x(x+h)}$$

$$f(x+h)-f(x)=\frac{-h}{x(x+h)}$$

$$\frac{f(x+h)-f(x)}{h}=\frac{-1}{x(x+h)}$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\frac{-1}{x^2}$$

or

6(b)(i)

$$y = \frac{1}{x}$$

$$y + \Delta y = \frac{1}{x + \Delta x}$$

$$\Delta y = \frac{1}{x + \Delta x} - \frac{1}{x}$$

$$= \frac{-\Delta x}{x(x + \Delta x)}$$

$$\frac{\Delta y}{\Delta x} = \frac{-1}{x(x + \Delta x)}$$

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{-1}{x^2}$$

Blunders (-3)

B1 $f(x+h)$

B2 Indices

B3 No limits shown or implied or no indication $\to 0$

B4 $h \to \infty$

Worthless

W1 Not 1st principles

(ii) Slope	**5 marks**	**Att 2**
Equation	**5 marks**	**Att 2**

6 (b) (ii) $\frac{dy}{dx} = \frac{-1}{x^2}$

At $\left(2, \frac{1}{2}\right)$, slope $= \frac{dy}{dx} = \frac{-1}{(2)^2} = \frac{-1}{4}$ $\therefore$ Tangent is line through $\left(2, \frac{1}{2}\right)$ with slope $m = \frac{-1}{4}$

$$y - y_1 = m(x - x_1)$$

$$y - \tfrac{1}{2} = \tfrac{-1}{4}(x - 2)$$

$$4y - 2 = -x + 2$$

$$x + 4y = 4$$

Blunders (-3)

B1 Differentiation

B2 Indices

B3 Equation formula line

B4 Substituting values into formula once only

Slips (-1)

S1 Numerical

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

6 (c) Let $f(x)=\tan^{-1}\frac{x}{2}$ and $g(x)=\tan^{-1}\frac{2}{x}$, for $x>0$.

(i) Find $f'(x)$ and $g'(x)$.

(ii) Hence, show that $f(x)+g(x)$ is constant.

(iii) Find the value of $f(x)+g(x)$.

(i) $f'(x)$	**5 marks**	**Att 2**
$g'(x)$	**5 marks**	**Att 2**
(ii)	**5 marks**	**Att 2**
(iii)	**5 marks**	**Att 2**

6 (c)(i) $f(x)=\tan^{-1}\left(\frac{x}{2}\right)$ [tables: $a=2$]

$$f'(x)=\frac{2}{4+x^2}$$

$$g(x)=\tan^{-1}\left(\frac{2}{x}\right)$$

$$g'(x)=\frac{1}{1+\left(\frac{2}{x}\right)^2}\cdot\left(-\frac{2}{x^2}\right)=\frac{-2}{4+x^2}$$

6(c)(ii) $f'(x)+g'(x)=\frac{2}{4+x^2}+\frac{-2}{4+x^2}=0$.

Derivative of $(f+g)$ is 0, so $(f+g)$ is constant.

or

6(c)(ii) $f'(x)+g'(x)=0\Rightarrow\int[f'(x)+g'(x)]dx=k$

$\Rightarrow f(x)+g(x)=k$ (constant)

6(c)(iii) Let $\tan^{-1}\left(\frac{x}{2}\right)+\tan^{-1}\left(\frac{2}{x}\right)=k$ $[x>0]$

Let $x=2$: $\tan^{-1}(1)+\tan^{-1}(1)=k$

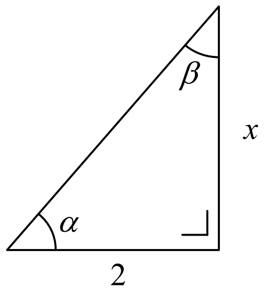
$$\frac{\pi}{4}+\frac{\pi}{4}=k$$

$$k=\frac{\pi}{2}$$

* Note: Any value of x in the domain can be used in place of $x=2$ above.

or

6(c)(iii)



$$\alpha + \beta = \frac{\pi}{2}$$

$$\tan \alpha = \frac{x}{2} \quad \Rightarrow \quad \alpha = \tan^{-1}\left(\frac{x}{2}\right)$$

$$\tan \beta = \frac{2}{x} \quad \Rightarrow \quad \beta = \tan^{-1}\left(\frac{2}{x}\right)$$

$$\tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{2}{x}\right) = \alpha + \beta = \frac{\pi}{2}$$

Blunders (-3)
B1 Differentiation
B2 Indices
B3 Integration

Slips (-1)
S1 Trig Value
S2 Numerical

QUESTION 7

Part (a)	10 marks	Att 3
Part (b)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)
Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

Part (a) **10 marks** **Att 3**

7 (a) Taking 1 as a first approximation of a root of $x^3 + 2x - 4 = 0$,
use the Newton Raphson method to calculate a second approximation of this root.

(a) **10 marks** **Att 3**

7 (a)

$f(x) = x^3 + 2x - 4$ $\qquad f'(x) = 3x^2 + 2$

$f(1) = (1)^3 + 2(1) - 4 = -1$ $\qquad f'(1) = 3(1)^2 + 2 = 5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \qquad x_1 = 1$$

$$= 1 - \frac{(-1)}{5} \qquad = 1 + \frac{1}{5} \qquad = \frac{6}{5}$$

Blunder (-3)
B1 Newton-Raphson formula once only
B2 Differentiation
B3 Indices
B4 $x_1 \neq 1$ once only

Slips (-1)
S1 Numerical
S2 Answer not tidied up

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7 (b) (i) Find the equation of the tangent to the curve $3x^2 + y^2 = 28$ at the point $(2, -4)$.

(ii) $x = e^t \cos t$ and $y = e^t \sin t$. Show that $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$.

(i) Differentiation	**5 marks**	**Att 2**
Equation	**5 marks**	**Att 2**

7 (b)(i)

$$3x^2 + y^2 = 28$$

$$6x + 2y\frac{dy}{dx} = 0$$

$$2y\left(\frac{dy}{dx}\right) = -6x$$

$$\frac{dy}{dx} = \frac{-6x}{2y} = \frac{-3x}{y}$$

At (2, -4), slope $= \frac{dy}{dx} = \frac{-3(2)}{-4} = \frac{3}{2}$

Tangent is line through (2, -4) with slope $m = \frac{3}{2}$

$$(y - y_1) = m(x - x_1)$$
$$y - (-4) = \frac{3}{2}(x - 2)$$
$$2(y + 4) = 3(x - 2)$$
$$2y + 8 = 3x - 6$$
$$3x - 2y - 14 = 0$$

Blunders (-3)
B1 Differentiation
B2 Incorrect values or no values
B3 Indices
B4 Equation of tangent
B5 Substituting values into formula once only

Slips (-1)
S1 Numerical

Worthless
W1 Integration
W2 No differentiation in 1st 5 marks

(ii) $\frac{dx}{dt}$ **and** $\frac{dy}{dt}$ **5 marks** **Att 2**

$\frac{dy}{dx}$ **5 marks** **Att 2**

7 (b)(ii)

$x = e^t(\cos t)$ $\quad\quad$ $y = e^t(\sin t)$

$$\frac{dx}{dt} = e^t(-\sin t) + \cos t(e^t) \quad\quad \frac{dy}{dt} = e^t(\cos t) + \sin t(e^t)$$
$$\frac{dx}{dt} = e^t \cos t - e^t \sin t \quad\quad \frac{dy}{dt} = e^t \cos t + e^t \sin t$$
$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{e^t \cos t + e^t \sin t}{e^t \cos t - e^t \sin t} = \frac{x + y}{x - y}$$

* Note**:** oversimplified differentiation in first 5 marks leads to Att 2 at most in second marks

Blunders (-3)
B1 Differentiation
B2 Indices
B3 Incorrect $\frac{dy}{dx}$
B4 Answer not in required form

Attempts
A1 Blunder in differentiation formula

Worthless
W1 Integration

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7 (c) $f(x) = \log_e 3x - 3x$, where $x > 0$.

(i) Show that $(\frac{1}{3}, -1)$ is a local maximum point of $f(x)$.

(ii) Deduce that the graph of $f(x)$ does not intersect the x-axis.

(i) Differentiation	**5 marks**	**Att 2**
Max Value	**5 marks**	**Att 2**
(ii) Only one root for $f'(x) = 0$	**5 marks**	**Att 2**
Absolute max pt.	**5 marks**	**Att 2**

7 (c)(i) $f(x) = \ln(3x) - 3x \qquad x > 0$

$$f'(x) = \frac{1}{3x}(3) - 3 \quad = \quad \frac{1}{x} - 3.$$

$$f''(x) = \frac{-1}{x^2}$$

Local max/min: $f'(x) = 0 \Rightarrow \frac{1}{x} - 3 = 0 \quad \Rightarrow \quad \frac{1}{x} = 3 \quad \Rightarrow \quad x = \frac{1}{3}.$

$$x = \frac{1}{3} \Rightarrow f''(x) = \frac{-1}{x^2} = \frac{-1}{\left(\frac{1}{3}\right)^2} < 0 \quad \Rightarrow \text{ local max at } x = \frac{1}{3}$$

$$x = \frac{1}{3} \quad \Rightarrow \quad f(x) = \ln(3x) - (3x) \quad = \ln(1) - (1) \quad = 0 - 1 \quad = -1 \Rightarrow \text{Local max at } \left(\frac{1}{3}, -1\right)$$

or

7(c)(i) $f(x) = \ln 3x - 3x$

$$f'(x) = \frac{1}{x} - 3$$

$$x = \frac{1}{3} \quad \Rightarrow \quad f'(x) = \frac{1}{\left(\frac{1}{3}\right)} - 3 = 3 - 3 = 0 \quad \Rightarrow \text{ turning pt at } x = \frac{1}{3}.$$

$$f''(x) = \frac{-1}{x^2} < 0 \text{ for all } x \quad \Rightarrow \quad \text{local max pt at} \quad x = \frac{1}{3}$$

$$x = \frac{1}{3} \quad \Rightarrow \quad y = \ln(3x) - 3x \quad = \ln(1) - 3\left(\frac{1}{3}\right) \quad = -1 \quad \Rightarrow \text{ local max is at } \left(\frac{1}{3}, -1\right)$$

(c)(ii) $f'(x)$ has only one root.
This implies that the local max. above is the only turning point.
And $f(x)$ is continuous, so the local max pt above is an absolute max. point.
Since max pt $\left(\frac{1}{3},-1\right)$ is below x-axis, the whole graph must lie below x-axis
Thus, $f(x)=0$ has no roots, since graph does not cut the x-axis.

* Accept work showing max point to be the only turning point and below x-axis, with or without a diagram.
* No need to mention "absolute" in answer.
* No need to mention continuity

Blunders (-3)
B1 Differentiation
B2 Not testing in $f''(x)$ for max
B3 Incorrect deduction or no deduction from test
B4 Incorrect y value or no y value
B5 Factors once only.

Slips (-1)
S1 $\ln 1 \neq 0$

Worthless
W1 No differentiation

QUESTION 8

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

8. **(a)** Find **(i)** $\int x^3 dx$ **(ii)** $\int \frac{1}{x^3}\, dx.$

(i)	**5 marks**	**Att2**
(ii)	**5 marks**	**Att 2**

Q8 (a) (i) $\int x^3 dx = \frac{x^4}{4} + c$

(ii) $\int \frac{1}{x^3}\, dx = \int x^{-3} dx = \frac{x^{-2}}{-2} + c = \frac{-1}{2x^2} + c$

* If c shown once, then no penalty

Blunders (-3)
B1 Integration
B2 Indices
B3 No *'c'* (penalize 1st integration)

Attempts
A1 Only *'c'* correct

Worthless
W1 Differentiation instead of integration

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

8 (b) (i) Evaluate $\int_0^4 x\sqrt{x^2+9}\, dx$.

(ii) f is a function such that $f'(x) = 6 - \sin x$ and $f\left(\frac{\pi}{3}\right) = 2\pi.$ Find $f(x)$.

(i) Integration	**5 marks**	**Att 2**
Value	**5 marks**	**Att 2**

8 (b) (i)

$$\int_0^4 x\sqrt{x^2+9}\, dx$$

$$= \int \left(\sqrt{x^2+9}\right) x dx$$

$$= \tfrac{1}{2}\int w^{\frac{1}{2}}.dw$$

$$= \tfrac{1}{2}\left[\frac{w^{\frac{3}{2}}}{\frac{3}{2}}\right] \quad = \tfrac{1}{3}\left[w^{\frac{3}{2}}\right]$$

$$= \tfrac{1}{3}\left[\left(x^2+9\right)^{\frac{3}{2}}\right]_0^4 \quad = \tfrac{1}{3}\left[(25)^{\frac{3}{2}} - (9)^{\frac{3}{2}}\right] \quad = \tfrac{1}{3}[125-27] = \frac{98}{3}$$

Let $w = x^2 + 9$

$$\frac{dw}{dx} = 2x$$

$$\frac{dw}{2} = xdx$$

* Incorrect substitution and unable to finish yields attempt at most.

Blunders (-3)

B1 Integration
B2 Indices
B3 Differentiation
B4 Limits
B5 Incorrect order in applying limits
B6 Not calculating substituted limits
B7 Not changing limits

Slips (-1)

S1 Answer not "tidied up".

Worthless

W1 Differentiation instead of integration except where other work merits attempt

(ii) $f(x)$	**5 marks**	**Att 2**
Value of c	**5 marks**	**Att 2**

8 (b) (ii)

$f'(x) = 6 - \sin x$

$f(x) = 6x + \cos x + c$

$f\left(\frac{\pi}{3}\right) = 6\left(\frac{\pi}{3}\right) + \cos\left(\frac{\pi}{3}\right) + c = 2\pi$

$2\pi + \frac{1}{2} + c = 2\pi$

$c = \frac{-1}{2}$

$\Rightarrow f(x) = 6x + \cos x - \frac{1}{2}$

Blunders (-3)

B1 Integration
B2 No *'c'*

Slips (-1)

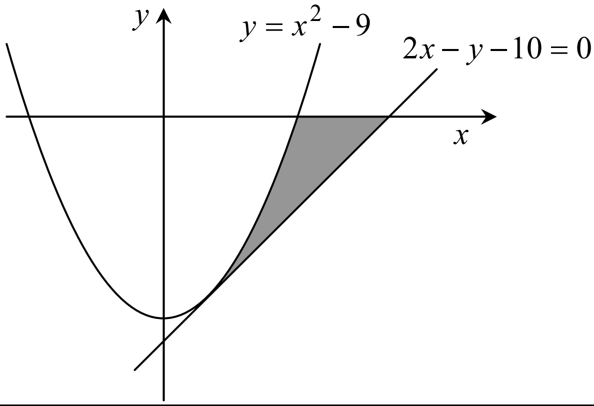
S1 Trig Value

Worthless

W1 Differentiation instead of integration, except where other work merits attempt.

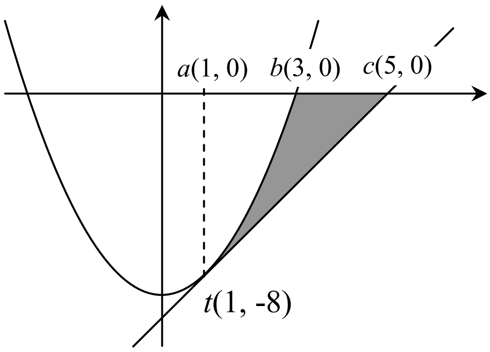
Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

8 (c) The line $2x - y - 10 = 0$ is a tangent to the curve $y = x^2 - 9$, as shown. The shaded region is bounded by the line, the curve and the x-axis. Calculate the area of this region.



Point (1, -8)	**5 marks**	**Att 2**
Points (3, 0) and (5, 0)	**5 marks**	**Att 2**
Area under curve between 1 and 3	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

8 (c)



$$\{y = 2x - 10\} \cap \{y = x^2 - 9\}$$
$$2x - 10 = x^2 - 9$$
$$0 = x^2 - 2x + 1$$
$$0 = (x-1)^2$$
$$\Rightarrow x = 1$$
$$y = 2(1) - 10$$
$$y = -8$$
$$\Rightarrow t(1, -8) \text{ and } a(1, 0)$$

Co-ords of b:

$$y = x^2 - 9$$
$$y = 0 : x^2 - 9 = 0$$
$$x^2 = 9$$
$$x = \pm 3$$
$$b(3, 0)$$

Co-ords of c:

$$y = 2x - 10$$
$$y = 0 : \ 0 = 2x - 10$$
$$x = 5$$
$$c(5, 0)$$

Shaded area = area Δact − area under curve

$|ac| = 4$ and $|at| = 8$ $\Rightarrow$ area $\Delta act = \frac{1}{2}|ac|.|at| = \frac{1}{2}(4)(8) = 16$

or

Area $\Delta act = \int_1^5 y dx = \int_1^5 (2x - 10) dx = \left[x^2 - 10x\right]_1^5 = |(25 - 50) - (1 - 10)| = |(-25) - (-9)| = 16.$

Area under curve $\int_1^3 y.dx = \int_1^3 (x^2 - 9) dx = \left[\frac{x^3}{3} - 9x\right]_1^3 = \left|(9 - 27) - \left(\frac{1}{3} - 9\right)\right| = \frac{28}{3}.$

Shaded area $= 16 - \frac{28}{3} = \frac{48 - 28}{3} = \frac{20}{3}$

Blunders (-3)

B1 Integration
B2 Indices
B3 Factors once only
B4 Calculation point of tangency
B5 Calculation of point where curve cuts x-axis
B6 Calculation of point where line cuts x-axis
B7 Error in area triangle
B8 Error in area formula
B9 Incorrect order in applying limits
B10 Not calculating substituted limits
B11 Error with line
B12 Error with curve
B13 Uses $\pi\int ydx$ for area formula

Attempts

A1 Uses volume formula
A2 Uses y^2 in formula

Worthless

W1 Differentiation instead of integration except where other work merits attempt
W2 Wrong area formula and no work

MARKING SCHEME

LEAVING CERTIFICATE EXAMINATION 2007

MATHEMATICS – HIGHER LEVEL – PAPER 2

GENERAL GUIDELINES FOR EXAMINERS – PAPER 2

1. Penalties of three types are applied to candidates' work as follows:
   - Blunders - mathematical errors/omissions (-3)
   - Slips - numerical errors (-1)
   - Misreadings (provided task is not oversimplified) (-1).

   Frequently occurring errors to which these penalties must be applied are listed in the scheme. They are labelled: B1, B2, B3,…, S1, S2,…, M1, M2,…etc. These lists are not exhaustive.

2. When awarding attempt marks, e.g. Att(3), note that
   - any *correct, relevant* step in a part of a question merits at least the attempt mark for that part
   - if deductions result in a mark which is lower than the attempt mark, then the attempt mark must be awarded
   - a mark between zero and the attempt mark is never awarded.

3. Worthless work is awarded zero marks. Some examples of such work are listed in the scheme and they are labelled as W1, W2,…etc.

4. The phrase "hit or miss" means that partial marks are not awarded – the candidate receives all of the relevant marks or none.

5. The phrase "and stops" means that no more work of merit is shown by the candidate.

6. Special notes relating to the marking of a particular part of a question are indicated by an asterisk. These notes immediately follow the box containing the relevant solution.

7. The sample solutions for each question are not intended to be exhaustive lists – there may be other correct solutions. Any examiner unsure of the validity of the approach adopted by a particular candidate to a particular question should contact his/her advising examiner.

8. Unless otherwise indicated in the scheme, accept the best of two or more attempts – even when attempts have been cancelled.

9. The *same* error in the *same* section of a question is penalised *once* only.

10. Particular cases, verifications and answers derived from diagrams (unless requested) qualify for attempt marks at most.

11. A serious blunder, omission or misreading results in the attempt mark at most.

12. Do not penalise the use of a comma for a decimal point, e.g. €5.50 may be written as €5,50.

QUESTION 1

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

1. **(a)** The following parametric equations define a circle:

$$x = 5 + 7\cos\theta, \quad y = 7\sin\theta, \text{ where } \theta \in \mathbf{R}.$$

What is the Cartesian equation of the circle?

Isolates 7cosθ	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

1 (a) $(x-5)^2 = 49\cos^2\theta$ and $y^2 = 49\sin^2\theta$.

$(x-5)^2 + y^2 = 49(\cos^2\theta + \sin^2\theta)$.

$\therefore\ (x-5)^2 + y^2 = 49$ or $x^2 + y^2 - 10x - 24 = 0$.

Blunders

B1 Error in transposition.

B2 Fails to square.

B3 $Sin^2\theta + \cos^2\theta \neq 1$.

Slips

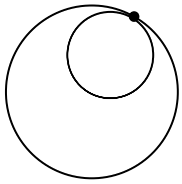
S1 Arithmetic errors.

Attempts

A1 Writes down equation of a circle without any further work.

Part (b) | **20 (5, 5, 5, 5) marks** | **Att (2, 2, 2, 2)**

1 (b) $x^2+y^2-4x-6y+5=0$ and $x^2+y^2-6x-8y+23=0$ are two circles.



(i) Prove that the circles touch internally.

(ii) Find the coordinates of the point of contact of the two circles.

(i) One centre and radius (same circle)	**5 marks**	**Att 2**
Distance between centres	**5 marks**	**Att 2**
Conclusion	**5 marks**	**Att 2**

1 (b) (i)

$x^2+y^2-4x-6y+5=0$ has centre $c_1(2,3)$ and radius $=r_1=\sqrt{4+9-5}=\sqrt{8}=2\sqrt{2}.$

$x^2+y^2-6x-8y+23=0$ has centre $c_2(3,4)$ and radius $=r_2=\sqrt{9+16-23}=\sqrt{2}.$

$|c_1c_2|=\sqrt{(2-3)^2+(3-4)^2}=\sqrt{2}.$

$r_1-r_2=2\sqrt{2}-\sqrt{2}=\sqrt{2}=|c_1c_2|.\quad \therefore$ Circles touch internally.

Blunders

B1 Error in finding centre or radius.
B2 Error in distance formula.
B3 Fails to show internal touching

Slips

S1 Arithmetic

(ii) | **5 marks** | **Att 2**

1 (b) (ii)

$$\begin{array}{r} x^2+y^2-4x-6y+5=0 \\ x^2+y^2-6x-8y+23=0 \\ \hline 2x+2y-18=0 \end{array}\quad \Rightarrow\ x+y-9=0\ \Rightarrow\ x=9-y.$$

$(9-y)^2+y^2-4(9-y)-6y+5=0\ \Rightarrow\ 81-18y+y^2+y^2-36+4y-6y+5=0$

$2y^2-20y+50=0\ \Rightarrow\ y^2-10y+25=0\ \Rightarrow\ (y-5)^2=0.\ \therefore y=5,\ x=4.$ Point is $(4,5)$.

* Accept correct answer without work.

Blunders

B1 Error in finding equation of the radical axis (common tangent)
B2 Error in substitution
B3 Error in solving quadratic

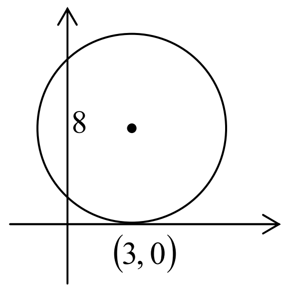
Slips

S1 Arithmetic error.

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

1 (c)

(c) A circle has its centre in the first quadrant.
The x-axis is a tangent to the circle at the point (3, 0).
The circle cuts the y-axis at points that are 8 units apart.
Find the equation of the circle.

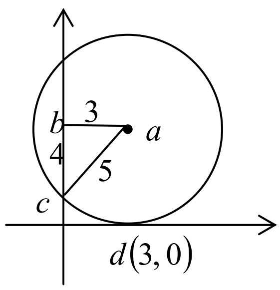


***x*-value of centre**	**5 marks**	**Att 2**
***ab* is perp bisector**	**5 marks**	**Att 2**
Radius	**5 marks**	**Att 2**
Equation	**5 marks**	**Att 2**

1 (c)

x value of centre is 3 as x-axis tangent at (3, 0).
ab is perpendicular bisector of chord, $\therefore |bc| = 4$.
Triangle abc is right-angled $\Rightarrow$ $ac = r = \sqrt{3^2 + 4^4} = 5$.
$|ac| = 5 = |ad| \Rightarrow y$ value of centre is 5.
Circle has centre (3, 5) and radius length 5.

$\therefore$ Circle: $(x-3)^2 + (y-5)^2 = 25$ or $x^2 + y^2 - 6x - 10y + 9 = 0$.



Blunders

B1 Incorrect x-coordinate of centre
B2 $|bc| \neq 4$
B3 Error in Pythagoras
B4 Error in forming equation of the circle.

Slips

S1 Arithmetic

QUESTION 2

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 marks** **Att 3**

2. **(a)** $\vec{x} = -2\vec{i} + 5\vec{j}$ and $\vec{xy} = -6\vec{i} - 8\vec{j}$. Express $\vec{y}$ in terms of $\vec{i}$ and $\vec{j}$.

(a) **10 marks** **Att 3**

2 (a) $\vec{x} + \vec{xy} = \vec{y} = -8\vec{i} - 3\vec{j}$.

Or $\vec{xy} = -6\vec{i} - 8\vec{j} \Rightarrow \vec{y} - \vec{x} = -6\vec{i} - 8\vec{j}$. $\therefore \vec{y} = -6\vec{i} - 8\vec{j} - 2\vec{i} + 5\vec{j} = -8\vec{i} - 3\vec{j}$.

Blunders

B1 $\vec{xy} \neq \vec{y} - \vec{x}$

B2 Error in transposing

Slips

S1 Arithmetic

Part (b) **20 (10, 10) marks** **Att (3, 3)**

(b) $\vec{a} = 5\vec{i}$ and $\vec{b} = \sqrt{3}\,\vec{i} + 3\vec{j}$.

(i) Show that $\vec{ab}$ is not perpendicular to $\vec{b}$.

(ii) Find the value of the real number k, given that $\vec{c} = k\vec{b}$ and $\vec{ac} \perp \vec{b}$.

(b) (i) **10 marks** **Att 3**

2 (b) (i)

$\vec{ab} = \vec{b} - \vec{a} = \sqrt{3}\,\vec{i} + 3\vec{j} - 5\vec{i} = \left(\sqrt{3} - 5\right)\vec{i} + 3\vec{j}$.

$\vec{ab} . \vec{b} = \sqrt{3}\left(\sqrt{3} - 5\right) + 9 \neq 0$. Not perpendicular.

Blunders

B1 $\vec{ab} \neq \vec{b} - \vec{a}$

B2 Error in transposing

B3 No conclusion

Slips

S1 Arithmetic

(b) (ii) **10 marks** **Att 3**

2 (b) (ii)

$$\vec{c} = k\vec{b} = \sqrt{3}k\,\vec{i} + 3k\,\vec{j}. \quad \vec{ac} = \vec{c} - \vec{a} = \left(\sqrt{3}k - 5\right)\vec{i} + 3k\,\vec{j}.$$

$$\vec{ac} \perp \vec{b} \;\Rightarrow\; \vec{ac}.\vec{b} = 0. \;\therefore\; \left(\sqrt{3}k - 5\right)\sqrt{3} + 9k = 0 \Rightarrow 12k = 5\sqrt{3} \Rightarrow k = \frac{5\sqrt{3}}{12}.$$

Blunders

B1 $\vec{ac} \neq \vec{c} - \vec{a}$

B2 Transposition errors

B3 No use of $\vec{ac}.\vec{b} = 0$

Slips

S1 Arithmetic

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

(c) $\vec{p} = 3\vec{i} + 4\vec{j}$ and $\vec{q} = 5\vec{i} + 12\vec{j}$.

$$\vec{r} = \frac{65t}{16}\left(\frac{\vec{p}}{|\vec{p}|} + \frac{\vec{q}}{|\vec{q}|}\right), \text{ where } t > 0.$$

(i) Express $\vec{r}$ in terms of $\vec{i}$ and $\vec{j}$.

(ii) Find $\vec{p}.\vec{r}$ and $\vec{q}.\vec{r}$.

(iii) Hence, show that r is on the bisector of $\angle poq$, where o is the origin.

Part (c) (i) **10 marks** **Att 3**

2 (c) (i)

$$\vec{r} = \frac{65t}{16}\left(\frac{\vec{p}}{|\vec{p}|} + \frac{\vec{q}}{|\vec{q}|}\right) = \frac{65t}{16}\left(\frac{3\vec{i} + 4\vec{j}}{5} + \frac{5\vec{i} + 12\vec{j}}{13}\right)$$

$$\vec{r} = \frac{65t}{16}\left(\frac{39\vec{i} + 52\vec{j} + 25\vec{i} + 60\vec{j}}{65}\right) = \frac{t}{16}\left(64\vec{i} + 112\vec{j}\right). \;\therefore\; \vec{r} = t\left(4\vec{i} + 7\vec{j}\right).$$

Blunders

B1 Error in $|\vec{p}|$ or $|\vec{q}|$

B2 Ignores t or t = some value.

S1 Arithmetic

Part (c) (ii) **5 marks** **Att 2**

2 (c) (ii)

$$\vec{p}.\vec{r} = \left(3\vec{i}+4\vec{j}\right)\left(4t\,\vec{i}+7t\,\vec{j}\right) = 12t+28t = 40t.$$

$$\vec{q}.\vec{r} = \left(5\vec{i}+12\vec{j}\right)\left(4t\,\vec{i}+7t\,\vec{j}\right) = 20t+84t = 104t.$$

Blunders

B1 Error in calculating scalar product

Slips

S1 Arithmetic

Part (c) (iii) **5 marks** **Att 2**

2 (c) (iii)

$$\vec{p}\,.\,\vec{r} = \left|\vec{p}\right|\left|\vec{r}\right|\cos\theta \Rightarrow 5t\sqrt{65}\cos\theta = 40t \Rightarrow \cos\theta = \frac{40}{5\sqrt{65}} = \frac{8}{\sqrt{65}}.$$

$$\vec{q}\,.\,\vec{r} = \left|\vec{q}\right|\left|\vec{r}\right|\cos\theta \Rightarrow 13t\sqrt{65} = 104t \Rightarrow \cos\theta = \frac{104}{13\sqrt{65}} = \frac{8}{\sqrt{65}}.$$

$\therefore\ r$ is on bisector of $\angle poq$.

Blunders

B1 Error in scalar product formula

B2 Error in modulus.

Slips

S1 Arithmetic errors

QUESTION 3

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 10) marks**	**Att (2, 2, 3)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a)	**10 marks**	**Att 3**

3. **(a)** Find the area of the triangle with vertices $(1,1), (8,-5)$ and $(5,-2)$.

(a)	**10 marks**	**Att 3**

3 (a)

Map $(1,1)$ onto $(0,0)$, $(8,-5)$ onto $(7,-6)$ and $(5,-2)$ onto $(4,-3)$.

Area of triangle $= \frac{1}{2}|x_1 y_2 - x_2 y_1| = \frac{1}{2}|-21+24| = \frac{3}{2}$.

or

Use $\frac{1}{2}$ base × perp. height:

Taking base:	[(1,1), (8,–5)]	[(1,1), (5,–2)]	[(8,–5), (5,–2)]
Length of base:	$\sqrt{85}$	5	$\sqrt{18}$
Equation base-line:	$6x+7y = 13$	$3x+4y = 7$	$x+y = 3$
Distance from other corner:	$\frac{3}{\sqrt{85}}$	$\frac{3}{5}$	$\frac{1}{\sqrt{2}}$
$\therefore$ Area =	3	3	3

Blunders

B1 Error in translation
B2 Error in formula for area of a triangle.
B3 Incorrect subst

Slips

S1 Arithmetic errors

Part (b) **20 (5, 5, 10) marks** **Att (2, 2, 3)**

> **3(b)** f is the transformation $(x, y) \to (x', y')$, where $x' = 4x + 2y$ and $y' = -3x - y$.
> K is the line $x + y = 0$.
> **(i)** Show that K is its own image under f.
> **(ii)** $p\ (1, -1)$ and $q\ (3, -3)$ are two points.
> Find the ratio $|pq| : |f(p)f(q)|$, giving your answer in its simplest form.

(i) Evaluate *x* and *y* **5 marks** **Att 2**
Find image **5 marks** **Att 2**

3 (b) (i)

$$x' = 4x + 2y$$
$$\underline{2y' = -6x - 2y}$$
$$x' + 2y' = -2x \;\Rightarrow\; x = \frac{1}{2}(-x' - 2y').$$
$$\text{But } y = \frac{1}{2}x' - 2x \Rightarrow y = \frac{1}{2}x' + x' + 2y' \Rightarrow y = \frac{1}{2}(3x' + 4y').$$
$$K : x + y = 0 \Rightarrow f(K) : \frac{1}{2}(-x' - 2y') + \frac{1}{2}(3x' + 4y') = 0$$
$$f(K) : -x' - 2y' + 3x' + 4y' = 0 \Rightarrow 2x' + 2y' = 0 \Rightarrow x' + y' = 0.$$
$$f(K) : x + y = 0 \;\Rightarrow\; f(K) = K.$$

Blunders
B1 Error in setting up or solving simultaneous equations.
B2 Error in finding image.

Slips
S1 Arithmetic errors

(b) (ii) **10 marks** **Att 3**

3 (b) (ii)

$$x' = 4x + 2y \text{ and } y' = -3x - y.$$
$$|pq| = \sqrt{(1-3)^2 + (-1+3)^2} = \sqrt{8} = 2\sqrt{2}.$$
$$f(p) = f(1, -1) = (2, -2) \text{ and } f(q) = f(3, -3) = (6, -6)$$
$$\Rightarrow\; |f(p)f(q)| = \sqrt{(2-6)^2 + (-2+6)^2} = \sqrt{32} = 4\sqrt{2}.$$
$$\therefore\; |pq| : |f(p)f(q)| = 2\sqrt{2} : 4\sqrt{2} = 1 : 2.$$

Blunders
B1 Error in distance formula
B2 Error in finding images
B3 No ratio shown or incorrect order.

Slips
S1 Arithmetic errors

Part (c) **20 (10, 5, 5) marks** **Att (3, 2, 2)**

3 (c) Consider the equation $k(3x-5y+6)+l(5x-7y+4)=0$.

(i) Show that, for any $k, l \in R$, the given equation represents a line passing through the point of intersection of $3x-5y+6=0$ and $5x-7y+4=0$.

(ii) Find the relationship between k and l for which the given equation represents a line of slope 2.

(iii) If $k=1$, what line through the point of intersection cannot be represented by the given equation? Justify your answer.

Part (c) (i) **10 marks** **Att 3**

3(c)(i) The given equation is of first degree in x and y and is therefore a line. It remains to show that it passes through the point of intersection.

Let (x_1, y_1) be the point of intersection of $3x-5y+6=0$ and $5x-7y+4=0$.

(x_1, y_1) is on the line $3x-5y+6=0 \Rightarrow 3x_1-5y_1+6=0$.

(x_1, y_1) is on the line $5x-7y+4=0 \Rightarrow 5x_1-7y_1+4=0$.

Blunders

B1 Fails to show expression represents a line.

B2 Fails to show passes through point of intersection.

Slips

S1 Arithmetic

Part (c) (ii) **5 marks** **Att 2**

3 (c) (ii) $k(3x-5y+6)+l(5x-7y+4)=0 \Rightarrow x(3k+5l)+y(-5k-7l)+(6k+4l)=0$

$\therefore$ Slope $=\dfrac{3k+5l}{5k+7l}=2 \Rightarrow 10k+14l=3k+5l \Rightarrow 7k+9l=0.$

Blunders

B1 Error in finding slope

B2 Transposing error

Slips

S1 Arithmetic errors.

Part (c) (iii) **5 marks** **Att 2**

3 (c) (iii)

If $k=1$, the equation $k(3x-5y+6)+l(5x-7y+4)=0$ cannot represent the line $5x-7y+4=0$.

Justification: If $k=1$, the slope of the line will be $\dfrac{3+5l}{5+7l}$.

There is no value of l that can make this expression equal to $\dfrac{5}{7}$,

(because attempting to solve this yields $21+35l=25+35l$, which has no solution).

Blunders

B1 Fails to justify answer.

QUESTION 4

Part (a)	**10(5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10(5,5) marks**	**Att (2, 2)**

4. (a) Show that $(\cos A + \sin A)^2 = 1 + \sin 2A.$

Square	**5 marks**	**Att 2**
Tidy up	**5 marks**	**Att 2**

4 (a)

$$(\cos A + \sin A)^2 = \cos^2 A + \sin^2 A + 2\cos A \sin A = 1 + \sin 2A.$$

Blunders

B1 $\cos^2 A + \sin^2 A \neq 1$

B2 $2\cos A \sin A \neq \sin 2A$

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

4 (b) Find all the solutions of the equation

$6\cos^2 x + \sin x - 5 = 0$, where $0^\circ \leq x \leq 360^\circ$.

Give the solutions correct to the nearest degree.

$\mathbf{\cos^2 x = 1 - \sin^2 x}$	**5 marks**	**Att 2**
Quadratic form	**5 marks**	**Att 2**
Solve quadratic	**5 marks**	**Att 2**
Values for *x*	**5 marks**	**Att 2**

4 (b)

$$6\cos^2 x + \sin x - 5 = 6(1 - \sin^2 x) + \sin x - 5 = 0 \Rightarrow 6\sin^2 x - \sin x - 1 = 0.$$

$$\therefore (2\sin x - 1)(3\sin x + 1) = 0 \Rightarrow \sin x = \frac{1}{2} \text{ or } \sin x = -\frac{1}{3}.$$

$$\therefore x = 30^\circ, 150^\circ, 199^\circ, 341^\circ.$$

Blunders

B1 Incorrect substitution for $\cos^2 x$

B2 Error in factors or quadratic formula.

B3 Each incorrect or missing solution.

Slips

S1 Arithmetic/not rounded.

Attempts

A1 $\cos^2 x = 1 - \sin^2 x$

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

4 (c) $[ab]$ is the diameter of a semicircle of centre o and radius-length r.

$[ac]$ is a chord such that $|\angle cab| = \alpha$, where α is in radian measure.

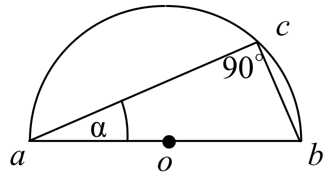
(i) Find $|ac|$ in terms of r and α.

(ii) $[ac]$ bisects the area of the semicircular region.

Show that $2\alpha + \sin 2\alpha = \dfrac{\pi}{2}$.

(c) (i)	**5 marks**	**Att 2**

4 (c) (i)



$[ab]$ is a diameter, $\therefore |\angle acb| = 90°$.

$\cos\angle cab = \cos\alpha = \dfrac{|ac|}{|ab|} = \dfrac{|ac|}{2r} \Rightarrow |ac| = 2r\cos\alpha$.

Blunders

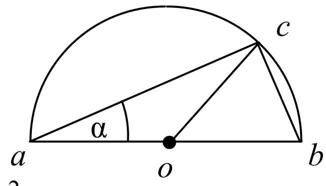
B1 Error in trig ratio

B2 $|ab| \neq 2r$

B3 Transposing error

(ii) Area of triangle	**5 marks**	**Att 2**
Area of sector	**5 marks**	**Att 2**
Show	**5 marks**	**Att 2**

4 (c) (ii)



Area of semicircle $= \dfrac{1}{2}\pi r^2$.

Area of region abc = Area of triangle aoc + sector obc.

Area of triangle $aoc = \dfrac{1}{2}|ao|.|ac|\sin\alpha = \dfrac{1}{2}r(2r\cos\alpha)\sin\alpha = \dfrac{1}{2}r^2\sin 2\alpha$.

Area of sector $obc = \dfrac{1}{2}r^2(2\alpha) = r^2\alpha$. as $|\angle cob| = 2\alpha$, since $|ao| = |oc| \Rightarrow |\angle aco| = \alpha$.

$\therefore$ Area of region $= r^2\alpha + \dfrac{1}{2}r^2\sin 2\alpha$.

This is half the semicircle, so $r^2\alpha + \dfrac{1}{2}r^2\sin 2\alpha = \dfrac{1}{4}\pi r^2 \Rightarrow 2\alpha + \sin 2\alpha = \dfrac{\pi}{2}$.

Blunders

B1 Error in area of triangle

B2 Error in area of sector

Slips

S1 Arithmetic

QUESTION 5

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 5,5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

5. **(a)** Evaluate $\displaystyle\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}$.

(a) **10 marks** **Att 3**

5 (a) $\displaystyle\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}=\lim_{x\to 0}\left(\frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}}\right)\times\frac{2}{3}==\frac{\lim_{x\to 0}\left(\frac{\sin 2x}{2x}\right)}{\lim_{x\to 0}\left(\frac{\sin 3x}{3x}\right)}\times\frac{2}{3}=\frac{1}{1}\times\frac{2}{3}=\frac{2}{3}.$

* Accept correct answer with or without work; if answer is correct, ignore the work.

Blunders

B1 sin2x = 2sinx.

B2 Error in differentiation

Slips

S1 Arithmetic

Part (b) **20 (10, 10) marks** **Att (3, 3)**

5 (b)

(b) Using the formula $\cos(A+B)=\cos A\cos B-\sin A\sin B$, derive a formula for $\cos(A-B)$ and hence prove that $\sin(A+B)=\sin A\cos B+\cos A\sin B$.

Formula for cos(*A*–*B*)	**10 marks**	**Att 3**
Hence prove	**10 marks**	**Att 3**

5 (b)

$\cos(A+B)=\cos A\cos B-\sin A\sin B \;\Rightarrow\; \cos(A-B)=\cos A\cos(-B)-\sin A\sin(-B).$

$\therefore \cos(A-B)=\cos A\cos B+\sin A\sin B$ as $\cos(-B)=\cos B$ and $\sin(-B)=-\sin B.$

$\sin(A+B)=\cos\left(90^\circ-[A+B]\right)=\cos\left([90^\circ-A]-B\right)$

$=\cos(90^\circ-A)\cos B+\sin(90^\circ-A)\sin B=\sin A\cos B+\cos A\sin B$

as $\cos(90^\circ-A)=\sin A$ and $\sin(90^\circ-A)=\cos A.$

Blunders

B1 Fails to replace B with -B

B2 Fails to show cos-B = cosB

B3 Fails to show sin -B = -sin B

B4 Hence not used

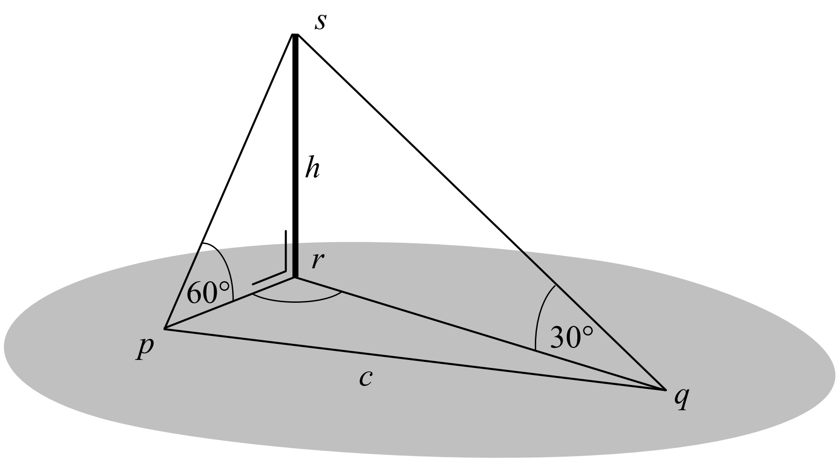
Slips

S1 Arithmetic error

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

5 (c)

(c) *p, q* and *r* are three points on horizontal ground.
[*sr*] is a vertical pole of height *h* metres.
The angle of elevation of *s* from *p* is 60° and the angle of elevation of *s* from *q* is 30°.
$|pq| = c$ metres.
Given that $3c^2 = 13h^2$, find $|\angle prq|$.



Calculates |*pr*|	**5 marks**	**Att 2**
Calculates |*rq*|	**5 marks**	**Att 2**
Cosine rule	**5 marks**	**Att 2**
Solves equation	**5 marks**	**Att 2**

5 (c)

$$\tan 60^\circ = \frac{h}{|pr|} \quad \Rightarrow \quad |pr| = \frac{h}{\tan 60^\circ} = \frac{h}{\sqrt{3}}.$$

$$\tan 30^\circ = \frac{h}{|rq|} \quad \Rightarrow \quad |rq| = \frac{h}{\tan 30^\circ} = \frac{h}{\frac{1}{\sqrt{3}}} = h\sqrt{3}.$$

$$\cos\angle prq = \frac{|pr|^2 + |rq|^2 - |pq|^2}{2|pr|.|rq|} = \frac{\frac{h^2}{3} + 3h^2 - c^2}{2h^2}$$

$$= \frac{10h^2 - 3c^2}{6h^2} = \frac{10h^2 - 13h^2}{6h^2} = -\frac{1}{2}. \qquad \therefore\ |\angle prq| = 120^\circ.$$

Blunders
B1 Error in trig ratio
B2 Error in Cosine Rule
B3 Error in solving equation

Slips
S1 Arithmetic errors.

QUESTION 6

Part (a)	**10 (5, 5) marks**	**Att (- , 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (- , 2)**

6. **(a)** Six people, including Mary and John, sit in a row.
(i) How many different arrangements of the six people are possible?
(ii) In how many of these arrangements are Mary and John next to each other?

(a) (i) **5 marks** **Hit/Miss**

6 (a) (i) Number of arrangements $= {}^6P_6 = 720.$

(a) (ii) **5 marks** **Att 2**

6 (a) (ii) Number of arrangements $= {}^5P_5 \times {}^2P_2 = 240.$

Blunders
B1 Fails to rearrange Mary and John
B2 Uses 5! + 2!

Slips
S1 Arithmetic

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

6 (b) α and β are the roots of the quadratic equation $px^2 + qx + r = 0.$
$u_n = l\alpha^n + m\beta^n$, for all $n \in$ N.
Show that $pu_{n+2} + qu_{n+1} + ru_n = 0$, for all $n \in$ N.

Uses root property correctly	**5 marks**	**Att 2**
Deduces u_{n+1}, u_{n+2}	**5 marks**	**Att 2**
Substitutes and tidies up	**5 marks**	**Att 2**
Conclusion	**5 marks**	**Att 2**

6 (b)

α is a root of $px^2 + qx + r = 0 \Rightarrow p\alpha^2 + q\alpha + r = 0$

$Similarly: p\beta^2 + q\beta + r = 0$

$Given: u_n = l\alpha^n + m\beta^n \Rightarrow u_{n+1} = l\alpha^{n+1} + m\beta^{n+1}, u_{n+2} = l\alpha^{n+2} + m\beta^{n+2}$

$\Rightarrow pu_{n+2} + qu_{n+1} + ru_n$

$= p\left[l\alpha^{n+2} + m\beta^{n+2}\right] + q\left[l\alpha^{n+1} + m\beta^{n+2}\right] + r\left[l\alpha^n + m\beta^n\right]$

$= l\alpha^n\left[p\alpha^2 + q\alpha + r\right] + m\beta^n\left[p\beta^2 + q\beta + r\right]$

$= l\alpha^n[0] + m\beta^n[0]$

$= 0$

Blunders

B1 Fails to use root property

B2 Error in expressing value of term

B3 Error in substituting or tidying

B4 No conclusion

Part (c)	**20 (10, 5, 5) marks**	**Att (3, 2, 2)**

6 (c) w white discs and r red discs are placed in a box. Two of the discs are drawn at random from the box. The probability that both discs are red is p..

(i) Find p in terms of w and r.

(ii) When $w = 1$, find the value of r for which $p = \frac{1}{2}$

(iii) There are other values of w and r that also give $p = \frac{1}{2}$.

The next smallest such value of w is even.
By investigating the even numbers in turn, find this value of w and the corresponding value of r.

(c) (i)	**10 marks**	**Att 3**

6 (c) (i) There are $(r + w)$ discs in the box.

$\therefore$ Number of ways of picking two discs $= {}^{r+w}C_2 = \frac{(r+w)(r+w-1)}{2}$.

Number of ways of picking two red discs $= {}^{r}C_2 = \frac{r(r-1)}{2}$.

$\therefore$ Probability $= \frac{r(r-1)}{(r+w)(r+w-1)}$.

Blunders

B1 Incorrect possible

B2 Incorrect favourable

B3 No division

Slips

S1 Arithmetic

(c) (ii)	**5 marks**	**Att 2**

6 (c) (ii)

$$\frac{r(r-1)}{(r+1)r} = \frac{1}{2} \Rightarrow 2(r-1) = r+1 \Rightarrow r = 3.$$

Blunders

B1 Error in solving the equation

Slips

S1 Arithmetic

(c) (iii) **5 marks** **Att 2**

6 (c) (iii)

$$\text{Probability} = \frac{r(r-1)}{(r+w)(r+w-1)} = \frac{1}{2}, \text{ for } w = 2, 4, 6, \text{etc.}$$

$$w = 2 \;\Rightarrow\; \frac{r(r-1)}{(r+2)(r+1)} = \frac{1}{2} \;\Rightarrow\; 2r^2 - 2r = r^2 + 3r + 2 \Rightarrow r^2 - 5r - 2 = 0.$$

No solution possible for r a natural number.

$$w = 4 \;\Rightarrow\; \frac{r(r-1)}{(r+4)(r+3)} = \frac{1}{2} \;\Rightarrow\; 2r^2 - 2r = r^2 + 7r + 12 \Rightarrow r^2 - 9r - 12 = 0.$$

No solution possible for r a natural number.

$$w = 6 \;\Rightarrow\; \frac{r(r-1)}{(r+6)(r+5)} = \frac{1}{2} \;\Rightarrow\; 2r^2 - 2r = r^2 + 11r + 30 \Rightarrow r^2 - 13r - 30 = 0.$$

$\therefore\ (r-15)(r+2) = 0 \;\Rightarrow\; r = 15$ as $r \neq -2$. $\therefore\ r = 15$ and $w = 6$.

Blunders

B1 Error in setting up or solving the equation

Slips

S1 Arithmetic

QUESTION 7

Part (a)	**10 (5, 5) marks**	**Att (- , 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5,5,5,5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att(- , 2)**

7.
(a) **(i)** How many different selections of four letters can be made from the letters of the word FLORIDA ?
(ii) How many of these selections contain at least one vowel?

(a) (i) **5 marks** **Hit/Miss**

7 (a) (i) $^7C_4 = 35.$

(a) (ii) **5 marks** **Att 2**

7 (a) (ii) Number of selections of four letters with no vowel $= {}^4C_4 = 1.$
Number of selections with at least one vowel $= 35 - 1 = 34.$

Blunders
B1 Error in deriving solution with no vowel
B2 Does not subtract

Part (b) **20 (10, 10) marks** **Att (3, 3)**

7 (b) Two dice are thrown.
(i) What is the probability of getting two identical numbers or a total of five?
(ii) What is the probability that the product of the two numbers thrown is at least twice their sum?

(b) (i) **10 marks** **Att 3**

7 (b) (i) Number of possible outcomes = 6×6 = 36.
Outcomes of interest: (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (1, 4), (4, 1), (2, 3), (3, 2).
So there are 10 outcomes of interest.
Probability = $\frac{10}{36} = \frac{5}{18}$.

Blunders
B1 Incorrect possible
B2 Incorrect favourable
B3 No division

Slips
S1 Arithmetic

(b) (ii) **10 marks** **Att 3**

7 (b) (ii)

Outcomes of interest	Product	Sum
(6, 6)	36	12
(6, 5) or (5, 6)	30	11
(6, 4) or (4, 6)	24	10
(6, 3) or (3, 6)	18	9
(5, 5)	25	10
(5, 4) or (4, 5)	20	9
(4, 4)	16	8

11 outcomes of interest: $\therefore$ Probability $= \dfrac{11}{36}$.

Blunders
B1 Incorrect possible
B2 Incorrect favourable
B3 No division

Slips
S1 Arithmetic

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

7 (c) (i) Find, in terms of a and d, the mean of the first seven terms of an arithmetic sequence with first term a and common ratio d.
(ii) Show that the standard deviation of these seven numbers is $2d$.

(c)(i) Correct Total **5 marks** **Att 2**
Mean **5 marks** **Att 2**

7 (c) (i) $S_7 = \frac{7}{2}(2a + 6d) = 7a + 21d \Rightarrow$ mean $= \bar{x} = \dfrac{7a+21d}{7} = a + 3d.$

Blunders
B1 Error in finding total
B2 Error in finding mean

Slips
S1 Arithmetic

(c)(ii) Correct total devs **5 marks** **Att 2**
Correct std dev **5 marks** **Att 2**

7 (c) (ii)

	a,	$a+d$,	$a+2d$,	$a+3d$,	$a+4d$,	$a+5d$,	$a+6d$
$\bar{x} = a + 3d \Rightarrow$	$-3d$,	$-2d$,	$-d$,	0,	d,	$2d$,	$3d$

$\sum d^2 = 28d^2$. $\therefore$ Standard deviation $\sqrt{\dfrac{28d^2}{7}} = 2d.$

Blunders
B1 Error in finding total
B2 Error in finding Std Dev

Slips
S1 Arithmetic

QUESTION 8

Part (a)	**10(5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**

8. (a) p and q are real numbers such that $p+q=1$.
Find the value of p that maximizes the product pq.

Expression in one variable	**5 marks**	**Att 2**
Finishes	**5 marks**	**Att 2**

8 (a)

$$pq = p(1-p) = p - p^2$$

$$\frac{d}{dp}\left(p-p^2\right) = 1-2p = 0 \text{ for maximum } \Rightarrow p = \frac{1}{2}.$$

$$\frac{d^2}{dp^2}\left(p-p^2\right) = -2 < 0 \Rightarrow \text{ maximum value at } p = \frac{1}{2}.$$

Blunders
B1 Error in finding expression
B2 Error in finding derivative

Slips
S1 Arithmetic

Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

8 (b) (i) Derive the Maclaurin series for $f(x)=(1+x)^m$ up to and including the term containing x^3.

(ii) Given that the general term of the series $f(x)$ is

$$\frac{m(m-1)(m-2).........(m-r+1)}{r!}x^r,$$

show that the series converges for $-1<x<1$.

(i) Differentiation	**5 marks**	**Att 2**
Evaluates at 0	**5 marks**	**Att 2**
Correct series	**5 marks**	**Att 2**

8 (b) (i)

$$f(x)=(1+x)^m \quad \Rightarrow \quad f(0)=1.$$

$$f'(x)=m(1+x)^{m-1} \quad \Rightarrow \quad f'(0)=m.$$

$$f''(x)=m(m-1)(1+x)^{m-2} \quad \Rightarrow \quad f''(0)=m(m-1).$$

$$f'''(x)=m(m-1)(m-2)m^{m-3} \quad \Rightarrow \quad f'''(0)=m(m-1)(m-2).$$

$$f(x)=f(0)+\frac{f'(0)x}{1!}+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+ \text{ ....}$$

$$\therefore f(x)=(1+x)^m=1+mx+\frac{m(m-1)x^2}{2!}+\frac{m(m-1)(m-2)x^3}{3!}+.....$$

Blunders
B1 Error in finding expression
B2 Error in finding Derivative
B3 Error in establishing series
Slips
S1 Arithmetic

Part (b) (ii) **5 marks** **Att 2**

8 (b) (ii)

$$u_{r+1} = \frac{m(m-1)(m-2).........(m-r+1)}{r!}x^r \quad \Rightarrow \quad u_r = \frac{m(m-1)(m-2).........(m-r)}{(r-1)!}x^{r-1}$$

$$\lim_{r\to\infty}\left|\frac{u_{r+1}}{u_r}\right| = \lim_{r\to\infty}\left|\frac{m(m-1)(m-2)........(m-r+1)}{r!}x^r \times \frac{(r-1)!}{m(m-1)(m-2).......(m-r)}.\frac{1}{x^{r-1}}\right|$$

$$= \lim_{r\to\infty}\left|\frac{(m-r+1)x}{r}\right| = \lim_{r\to\infty}\left|\frac{mx}{r} - x + \frac{x}{r}\right| = |-x| \Rightarrow |x| < 1 \text{ for convergency.}$$

$$|x| < 1 \quad \Rightarrow \quad -1 < x < 1.$$

Blunders
B1 Error in finding expression
B2 Error in finding Derivative
B3 Error in establishing range.

Slips
S1 Arithmetic

Part (c)	20 (5, 5, 5, 5) marks	Att (2, 2, 2, 2)

8 (c) Evaluate $\int_0^1 \tan^{-1} x dx.$

Set up integration by parts	**5 marks**	**Att 2**
Parts stage done	**5 marks**	**Att 2**
$\int \frac{x}{1+x^2} dx$	**5 marks**	**Att 2**
Evaluation	**5 marks**	**Att 2**

8 (c)

$$\int u dv = uv - \int v du. \quad \text{Let } u = \tan^{-1} x \text{ and } dv = dx. \ \therefore du = \frac{1}{1+x^2} dx \text{ and } v = x.$$

$$\therefore \int_0^1 \tan^{-1} x dx = uv - \int v du = x \tan^{-1} x - \int_0^1 \frac{x}{1+x^2} dx.$$

$$\int_0^1 \frac{x}{1+x^2} dx. \quad \text{Let } w = 1 + x^2 \Rightarrow dw = 2x dx.$$

$$\therefore \int_0^1 \frac{x dx}{1+x^2} = \frac{1}{2} \int_1^2 \frac{dw}{w} = \frac{1}{2} [\log_e x]_1^2 = \frac{1}{2} [\log_e 2 - \log_e 1] = \frac{1}{2} \log_e 2.$$

$$\therefore \int_0^1 \tan^{-1} x dx = [x \tan^{-1} x]_0^1 - \frac{1}{2} \log_e 2 = \tan^{-1} 1 - \frac{1}{2} \log_e 2 = \frac{\pi}{4} - \frac{1}{2} \log_e 2.$$

Blunders

B1 Error in setting up expression
B2 Error in integrating
B3 Error in establishing value

Slips

S1 Arithmetic

Note: Candidates may attempt to use a Maclaurin expansion to answer this question. They are unlikely to make substantial progress. The solution below is presented so as to facilitate the award of relevant partial credit.

Expand $\tan^{-1}x$	**5 marks**	**Att 2**
Integrate & evaluate at limits	**5 marks**	**Att 2**
Partial fractions & separate 2 series	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots$$

$$\therefore \int_0^1 \tan^{-1} x dx = \frac{x^2}{1.2} - \frac{x^4}{3.4} + \frac{x^6}{5.6} - \frac{x^8}{7.8} + \cdots + (-1)^n \frac{x^{2n+2}}{(2n+1)(2n+2)} + \cdots$$

$$= \frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \cdots + (-1)^n \frac{1}{(2n+1)(2n+2)} + \cdots$$

$$= \left(\frac{1}{1} - \frac{1}{2}\right) - \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{5} - \frac{1}{6}\right) - \left(\frac{1}{7} - \frac{1}{8}\right) + \cdots + (-1)^n \left(\frac{1}{2n+1} - \frac{1}{2n+2}\right) + \cdots$$

$$= \left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots + \frac{(-1)^n}{2n+1} + \cdots\right) - \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{6} - \frac{1}{8} + \cdots + \frac{(-1)^n}{2n+2} + \cdots\right)$$

$$= \tan^{-1} 1 - \frac{1}{2}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \frac{(-1)^n}{n+1} + \cdots\right)$$

$$= \tan^{-1} 1 - \frac{1}{2}\ln(1+1)$$

$$= \frac{\pi}{4} - \frac{1}{2}\ln 2 .$$

QUESTION 9

Part (a)	**10 (5, 5) marks**	**Att (2, 2)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 (5, 5) marks** **Att (2, 2)**

9. **(a)** Two events E_1 and E_2 are independent. $P(E_1) = \frac{1}{5}$ and $P(E_2) = \frac{1}{7}$. Find

(i) $P(E_1 \cap E_2)$

(ii) $P(E_1 \cup E_2)$.

(a) (i) **5 marks** **Att 2**

9 (a) (i) $P(E_1 \cap E_2) = P(E_1).P(E_2) = \frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$.

Blunders

B1 Addition for multiplication

Slips

S1 Arithmetic

(a) (ii) **5 marks** **Att 2**

9 (a) (ii) $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) = \frac{1}{5} + \frac{1}{7} - \frac{1}{35} = \frac{11}{35}$.

Blunders

B1 Multiplication for addition

B2 Double counts

Slips

S1 Arithmetic

Part (b) **20 (10, 10) marks** **Att (3, 3)**

9 (b) Five unbiased coins are tossed. **(i)** Find the probability of getting three heads and two tails. **(ii)** The five coins are tossed eight times. Find the probability of getting three heads and two tails exactly four times. Give your answer correct to three places of decimals.

Part (b) (i) **10 marks** **Att 3**

9 (b) (i) $p=\frac{1}{2},\ q=\frac{1}{2} \Rightarrow \text{Probability}={}^5C_3\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^2=\frac{10}{32}=\frac{5}{16}.$

Blunders

B1 Error in finding *p or q*

B2 Error in finding Binomial Coefficients

B3 Error in evaluation

Slips

S1 Arithmetic

Part (b) (ii) **10 marks** **Att 3**

9 (b) (ii) $p=\frac{5}{16},\ q=\frac{11}{16} \Rightarrow \text{Probability}={}^8C_4\left(\frac{5}{16}\right)^4\left(\frac{11}{16}\right)^4=0.149.$

Blunders

B1 Error in finding p *or q*

B2 Error in binomial coefficients

Slips

S1 Arithmetic

Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

9 (c) The amounts due on monthly mobile phone bills in Ireland are normally distributed with mean €53 and standard deviation €15.

(i) If a monthly phone bill is chosen at random, find the probability that the amount due is between €47 and €74.

(ii) A random sample of 900 mobile phone bills is taken. Find the probability that the mean amount due on the bills in the sample is greater than €53·3.

(i) z_1 and z_2	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

9 (c) (i)

$\bar{x} = 53, \ \sigma = 15. \quad P(47 < x < 74) = P(z_1 < z < z_1).$

$z_1 = \frac{x - \bar{x}}{\sigma} = \frac{47 - 53}{15} = -\frac{6}{15} = -0.4. \qquad z_2 = \frac{74 - 53}{15} = \frac{21}{15} = 1.4.$

$P(-0.4 < z < 1.4) = P(z \le 1.4) - P(z > 0.4) = 0.9192 - [1 - P(z \le 0.4)]$

$= 0.9192 - (1 - 0.6554) = 0.9192 - 0.3446 = 0.5746.$

Blunders

B1 Error in finding z_1 or z_2

B2 Error in setting up probability

B3 Error in evaluation

Slips

S1 Arithmetic

(ii) Std error	**5 marks**	**Att 2**
Finish	**5 marks**	**Att 2**

9 (c) (ii)

$n = 900, \ \bar{x} = 53, \ \sigma = 15.$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{900}} = \frac{15}{30} = \frac{1}{2}$

$P(x > 53.3) = P\left(z > \frac{53.3 - 53}{0.5}\right) = P(z > 0.6) = 1 - P(z \le 0.6)$

$= 1 - 0.7257 = 0.2743.$

Blunders

B1 Error in finding standard error

B2 Error in evaluation

Slips

S1 Arithmetic

QUESTION 10

Part (a)	**10 (5, 5) marks**	**Att (- , -)**
Part (b)	**20 (10, 10) marks**	**Att (3, 3)**
Part (c)	**20 (10, 10) marks**	**Att (3, 3)**

Part (a) **10 (5, 5) marks** **Att (- , -)**

10. (a) For each of the following, give a reason why it is not a group.

(i) The set of natural numbers under subtraction.

(ii) The set of real numbers under multiplication.

(a) (i) **5 marks** **Hit/Miss**

10 (a) (i) Not closed: e.g. $6-14=-8,\ -8 \notin \mathbf{N}$.

(a) (ii) **5 marks** **Hit/Miss**

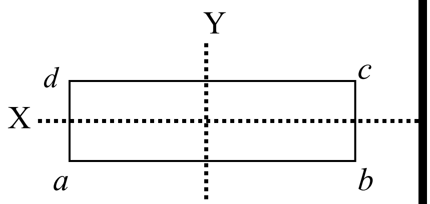
10 (a) (ii) Not all elements have inverses: $0 \in \mathbf{R}$, but 0 has no multiplicative inverse in **R**.

Part (b) **20 (10, 10) marks** **Att (3, 3)**

10 (b) $G=\{I_\pi, R_{180^\circ}, S_X, S_Y\}$ is the set of symmetries of the rectangle *abcd*.

(i) Show that G is a group under composition.
You may assume that composition is associative.

(ii) Find $Z(G)$, the centre of the group.



Part (b) (i) **10 marks** **Att 3**

10 (b) (i)

o	I_π	R_{180°	S_X	S_Y
I_π	I_π	R_{180°	S_X	S_Y
R_{180°	R_{180°	I_π	S_Y	S_X
S_X	S_X	S_Y	I_π	R_{180°
S_Y	S_Y	S_X	R_{180°	I_π

Closed: No new element.

Associative: yes, given.

Identity: I_π

Inverses: $(I_\pi)^{-1}=I_\pi,\ (R_{180^\circ})^{-1}=R_{180^\circ},$

$(S_X)^{-1}=S_X,\ (S_Y)^{-1}=S_Y.$

Blunders

B1 Identity not given
B2 Inverses not stated
B3 Closure not defined

Slips

S1 each inverse not given

Part (b) (ii) **10 marks** **Att 3**

10 (b) (ii) In table elements are symmetrical about main diagonal.

$\therefore$ G is a commutative group $\Rightarrow$ $G(Z) = G = \{I_\pi, R_{180^\circ}, S_X, S_Y\}$.

or from the table, x o y = y o x for all x,y $\in G$ i.e each element commutes with each other element so Z(G) = G.

Blunders

B1 Each element missing from set.

Part (c) **20 (10, 10) marks** **Att (3, 3)**

10 (c) Use Lagrange's theorem to prove that

(i) any group of prime order is cyclic.

(ii) the order of any element of a finite group *G* divides the order of *G*.

Part (c) (i) **10 marks** **Att 3**

10 (c) (i) Let $(G,*)$ be a group of order *k*, where *k* is prime.

Let $a \in G$ and $a \neq e$. $\therefore \langle a \rangle$, (the group generated by *a*,) is a subgroup of *G*.

Hence, the order of $\langle a \rangle$ is a factor of *k* (by Lagrange's theorem).

But *k* is prime $\Rightarrow$ order of $\langle a \rangle = k$, (since the order of $\langle a \rangle \neq 1$, since $a \neq e$).

$\therefore \langle a \rangle = G$. $\therefore$ G is cyclic.

Blunders

B1 Fails to establish that $< a >$ is a subgroup of *G*.

B2 Fails to use Lagrange

B3 No conclusion

Part (c) (ii) **10 marks** **Att 3**

10 (c) (ii) Let $(G,*)$ be a group of order *n*.

Let $a \in G$ and let the order of a be m.

Then *m* is also the order of $<a>$, the subgroup generated by *a*.

But the order of a subgroup divides the order of the group (by Lagrange's theorem).

$\therefore$ *m* is a factor of *n*.

That is, the order of the element *a* divides the order of the group *G*.

Blunders

B1 Fails to use Lagrange

B2 No conclusion.

QUESTION 11

Part (a)	**10 marks**	**Att 3**
Part (b)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**
Part (c)	**20 (5, 5, 5, 5) marks**	**Att (2, 2, 2, 2)**

Part (a) **10 marks** **Att 3**

11. (a) Find the eccentricity of an ellipse with equation $\frac{x^2}{64}+\frac{y^2}{48}=1$.

(a) **10 marks** **Att 3**

11 (a) $a^2=64,\ b^2=48$ and $b^2=a^2(1-e^2)$.

$48=64(1-e^2)\Rightarrow 64e^2=16\Rightarrow e^2=\frac{1}{4}.\ \therefore e=\frac{1}{2}.$

Blunders

B1 Incorrect a^2

B2 $b^2\neq a^2(1-e^2)$

Slips

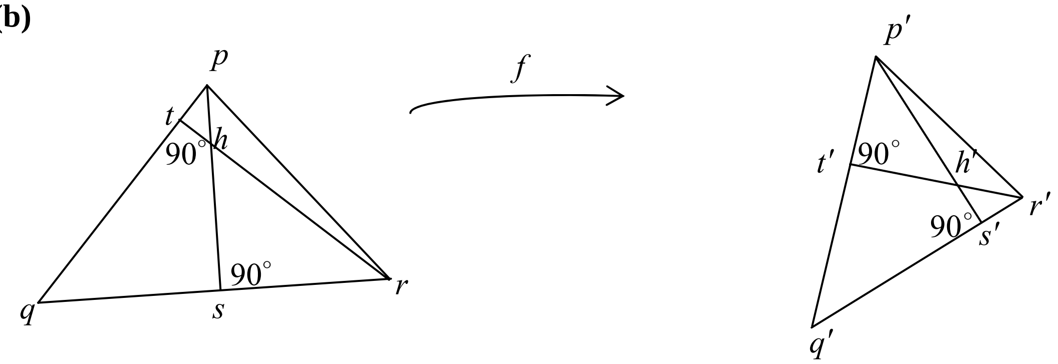
S1 Arithmetic

Part (b) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

11 (b) Prove that a similarity transformation maps the orthocentre of a triangle onto the orthocentre of the image triangle.

Orthocentre	**5 marks**	**Att 2**
Mapping	**5 marks**	**Att 2**
Perp invariant	**5 marks**	**Att 2**
Conclusion	**5 marks**	**Att 2**

11 (b)



f is a similarity transformation.

$[ps]\perp[qr]$ and $[rt]\perp[pq]\ \Rightarrow\ h$ is the orthocentre of triangle pqr.

By f, triangle pqr is mapped to triangle $p'q'r'$.

To Prove: $f(h)$ is orthocentre of triangle $p'q'r'$.

By f, $[ps]$ maps to $[p's']$ and $[rt]$ maps to $[r't']$.

But $[ps]\perp[qr]\ \Rightarrow\ [p's']\perp[q'r']$ as perpendicularity is invariant.

Similarly $[r't']\perp[p'q']$. $\therefore\ h'$ is orthocentre of triangle $p'q'r'$.

But $h=[ps]\cap[rt]$ maps to $f(h)=[p's']\cap[r't']$.

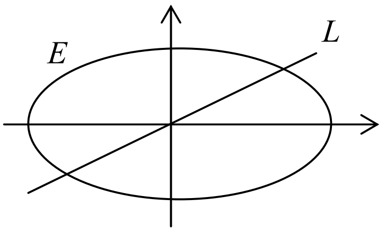
$\therefore\ f(h)=h'\ \Rightarrow\ f(h)$ is orthocentre of triangle $p'q'r'$.

Blunders
B1 Fails to define orthocentre
B2 Fails to state perpendicularity is invariant
B3 Fails to show h maps onto f(h)

Part (c) **20 (5, 5, 5, 5) marks** **Att (2, 2, 2, 2)**

11 (c) E is the ellipse $\frac{x^2}{4} + y^2 = 1$ and L is the line $y = x$.

Using a transformation that maps E to the unit circle, or otherwise, find the equation of the diameter that is conjugate to L in E.



Transformation	**5 marks**	**Att 2**
***f*(*L*)**	**5 marks**	**Att 2**
Conjugate diameter	**5 marks**	**Att 2**
Inverse map	**5 marks**	**Att 2**

11 (c)

Let f be the transfomation $(x, y) \to (x', y')$ where $x = 2x'$ and $y = y'$.

$f(E): \frac{(2x')^2}{4} + (y')^2 = 1 \Rightarrow (x')^2 + (y')^2 = 1.$

L is $x - y = 0 \Rightarrow f(L): 2x' - y' = 0.$

But $f(E)$ is a circle, so the conjugate diameter is the perpendicular diameter.

Slope of $f(L)$ is 2, so slope of $f(K)$ is $-\frac{1}{2}$. Hence, equation of $f(K)$ is $x' + 2y' = 0.$

By f^{-1}, $f(K)$ maps to K, the conjugate diameter of L in E.

Applying f^{-1}, we get $K: \frac{x}{2} + 2y = 0$.

So, the conjugate diameter of L is $x + 4y = 0.$

Blunders
B1 Incorrect transformation
B2 Incorrect image for x - y =0
B3 Error in mapping back to the ellipse.

Slips
S1 Arithmetic

Marcanna Breise as ucht freagairt trí Ghaeilge

(Bonus marks for answering through Irish)

Ba chóir marcanna de réir an ghnáthráta a bhronnadh ar iarrthóirí nach ngnóthaíonn thar 75% d'iomlán na marcanna don pháipéar. Ba chóir freisin an marc bónais sin a shlánú **síos**.

Déantar an cinneadh agus an ríomhaireacht faoin marc bónais i gcás gach páipéar ar leithligh.

Is é 5% an gnáthráta agus is é 300 iomlán na marcanna don pháipéar. Mar sin, bain úsáid as an ngnáthráta 5% i gcás marcanna suas go 225. (e.g. 198 marks × 5% = 9·9 ⇒ bónas = 9 marc.)

Thar 225, is féidir an bónas a ríomh de réir na foirmle seo: [300 – bunmharc] × 15%, (agus an marc sin a shlánú **síos**). In ionad an ríomhaireacht sin a dhéanamh, is féidir úsáid a bhaint as an tábla thíos.

Bunmharc	Marc Bónais
226	11
227 – 233	10
234 – 240	9
241 – 246	8
247 – 253	7
254 – 260	6
261 – 266	5
267 – 273	4
274 – 280	3
281 – 286	2
287 – 293	1
294 – 300	0