Coimisiún na Scrúduithe Stáit

State Examinations Commission

Leaving Certificate Examination 2017

Mathematics

Paper 2

Higher Level

Monday 12 June Morning 9:30 – 12:00

300 marks

Examination number

Centre stamp

Running total

For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade

Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer **all nine** questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

You will lose marks if you do not show all necessary work.

You may lose marks if you do not include appropriate units of measurement, where relevant.

You may lose marks if you do not give your answers in simplest form, where relevant.

Write the make and model of your calculator(s) here: [ ]

Section A — Concepts and Skills — 150 marks

Answer **all six** questions from this section.

Question 1 **(25 marks)**

When Conor rings Ciara's house, the probability that Ciara answers the phone is $\frac{1}{5}$.

(a) Conor rings Ciara's house once every day for 7 consecutive days. Find the probability that she will answer the phone on the 2nd, 4th, and 6th days but not on the other days.

(b) Find the probability that she will answer the phone for the 4th time on the 7th day.

(c) Conor rings her house once every day for *n* days. Write, in terms of *n*, the probability that Ciara will answer the phone at least once.

(d) Find the minimum value of *n* for which the probability that Ciara will answer the phone at least once is greater than 99%.

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Question 2 **(25 marks)**

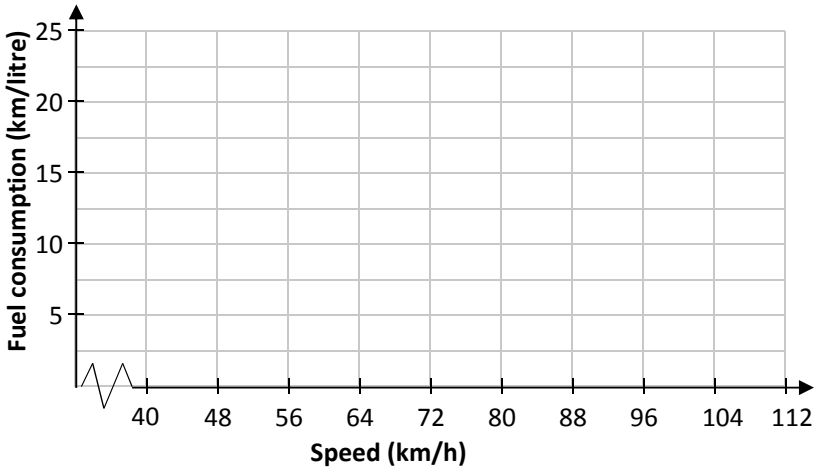
An experiment measures the fuel consumption at various speeds for a particular model of car. The data collected are shown in Table 1 below.

Table 1							
Speed (km/hour)	40	48	56	64	88	96	112
Fuel consumption (km/litre)	21	16	18	16	13	11	9

(a) Find the correlation coefficient of the data in Table 1, correct to 3 decimal places.

Correlation Coefficient $=$	

(b) Plot the points from the table on the grid below **and** draw the line of best fit (by eye).



(c) The slope of the line of best fit is found to be $-0{\cdot}15$.
What does this value represent in the context of the data?

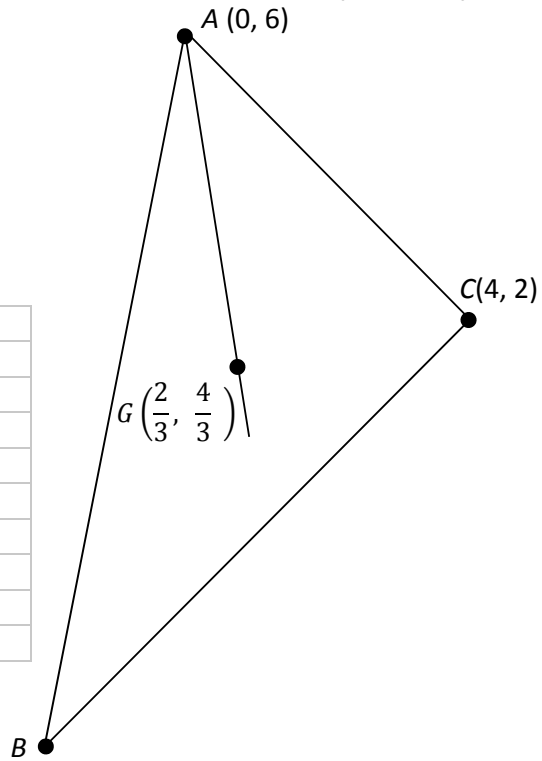
(d) Mary drove from Cork to Dublin at an average speed of 96 km/h.
Jane drove the same journey at an average speed of 112 km/h.
Each travelled 260 km and paid 132·9 cents per litre for the fuel.
Both used the model of car used to generate the data in Table 1.

(i) Find how much longer it took Mary to complete the journey.
Give your answer correct to the nearest minute.

(ii) Based on the data in Table 1 and their average speeds, find how much more Jane spent on fuel during the course of this journey.

Question 3 **(25 marks)**

ABC is a triangle where the co-ordinates of *A* and *C* are (0, 6) and (4, 2) respectively.
$G\left(\frac{2}{3}, \frac{4}{3}\right)$ is the centroid of the triangle *ABC*.
AG intersects *BC* at the point *P*.
$|AG| : |GP| = 2 : 1$.



(a) Find the co-ordinates of *P*.

(b) Find the co-ordinates of *B*.

(c) Prove that *C* is the orthocentre of the triangle *ABC*.

Question 4 **(25 marks)**

$A(0, 0)$, $B(6{\cdot}5, 0)$ and $C(10, 7)$ are three points on a circle.

(a) Find the equation of the circle.

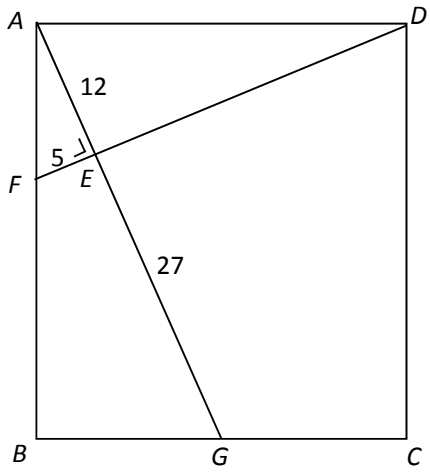
(b) Find $|\angle BCA|$. Give your answer in degrees, correct to 2 decimal places.

Question 5 **(25 marks)**

ABCD is a rectangle.

$F \in [AB]$, $G \in [BC]$, $[FD] \cap [AG] = \{E\}$, and $FD \perp AG$.

$|AE| = 12$ cm, $|EG| = 27$ cm, and $|FE| = 5$ cm.



(a) Prove that ΔAFE and ΔDAE are similar (equiangular).

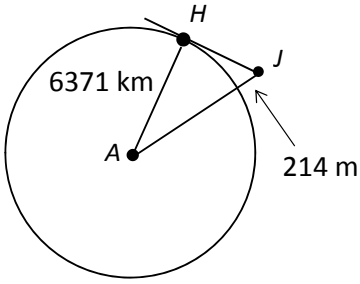
(b) Find $|AD|$.

(c) ΔAFE and ΔAGB are similar. Show that $|AB| = 36$ cm.

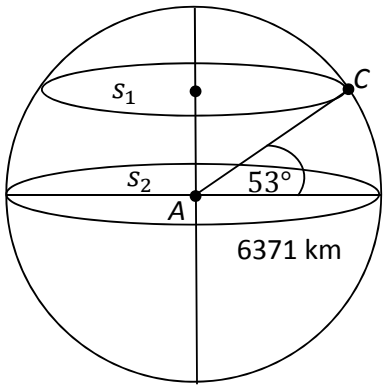
(d) Find the area of the quadrilateral *GCDE*.

Question 6 **(25 marks)**

(a) Take the earth as a sphere with radius 6371 km.
Jack is standing on the Cliffs of Moher at the point *J* which is 214 metres above sea level.
He is looking out to sea at a point *H* on the horizon.
Taking *A* as the centre of the earth, find $|JH|$, the distance from Jack to the horizon.
Give your answer correct to the nearest km.



(b) The Cliffs of Moher, at point *C*, are at latitude $53°$ north of the equator.
On the diagram, s_1 represents the circle that is at latitude 53°.
s_2 represents the equator (which is at latitude $0°$).
A is the centre of the earth.
s_1 and s_2 are on parallel planes.
Find the length of the circle s_1.
Give your answer correct to the nearest km.



Section B — Contexts and Application — 150 marks

Answer **all three** questions from this section.

Question 7 **(40 marks)**

Two solid cones, each of radius R cm and height R cm are welded together at their vertices and placed in the smallest possible hollow cylinder, as shown in **Figure 1** below.

Figure 1

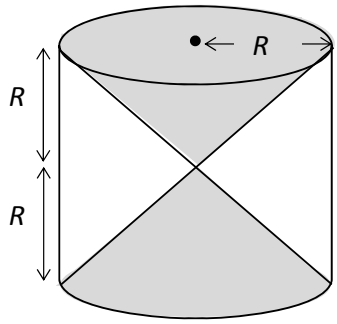
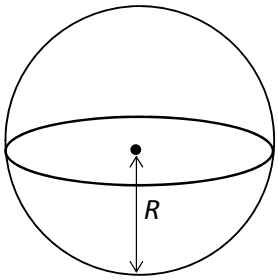


Figure 2



(a) Show that the capacity (volume) of the empty space in the cylinder is equal to the capacity of an empty sphere of radius R cm (**Figure 2**).

(b) In the remainder of this question, $R = 12$ cm. Water is poured into both the cylinder and the sphere to a depth of 6 cm as shown below (**Figure 3** and **Figure 4** respectively).

(i) Find $|AB|$, the radius of the circular surface of the water in the sphere (**Figure 4**). Give your answer in the form $a\sqrt{b}$ cm, where $a, b \in \mathbb{N}$.

Figure 3

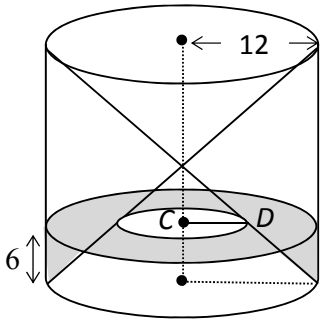
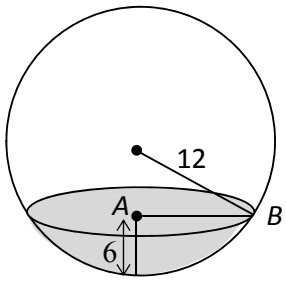


Figure 4



(ii) Find $|CD|$, the radius of the cone at water level, as shown in **Figure 3**.

(iii) Verify that the area of the surface of the water in the sphere is equal to the area of the surface of the water in the cylinder.

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(c) The mathematician Cavalieri discovered that, at the same depth, the volume of water in the available space in the cylinder is equal to the volume of water in the sphere.
Use this discovery to find the volume of water in the sphere when the depth is 6 cm.
Give your answer in terms of π.

Question 8 **(60 marks)**

(a) In 2015, in a particular country, the weights of 15 year olds were normally distributed with a mean of 63·5 kg and a standard deviation of 10 kg.

(i) In 2015, Mariska was a 15 year old in that country. Her weight was 50 kg.
Find the percentage of 15 year olds in that country who weighed more than Mariska.

(ii) In 2015, Kamal was a 15 year old in that country.
1·5% of 15 year olds in that country were heavier than Kamal.
Find Kamal's weight.

(iii) In 2016, 150 of the 15 year olds in that country were randomly selected and their weights recorded. It was found that their weights were normally distributed with a mean weight of 62 kg and a standard deviation of 10 kg. Test the hypothesis, at the 5% level of significance, that the mean weight of 15 year olds, in that country, had not changed from 2015 to 2016. State the null hypothesis and your alternative hypothesis. Give your conclusion in the context of the question.

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(b) In Galway, rain falls in the morning on $\frac{1}{3}$ of the school days in the year.

When it is raining the probability of heavy traffic is $\frac{1}{2}$.

When it is not raining the probability of heavy traffic is $\frac{1}{4}$.

When it is raining and there is heavy traffic, the probability of being late for school is $\frac{1}{2}$.

When it is not raining and there is no heavy traffic, the probability of being late for school is $\frac{1}{8}$.

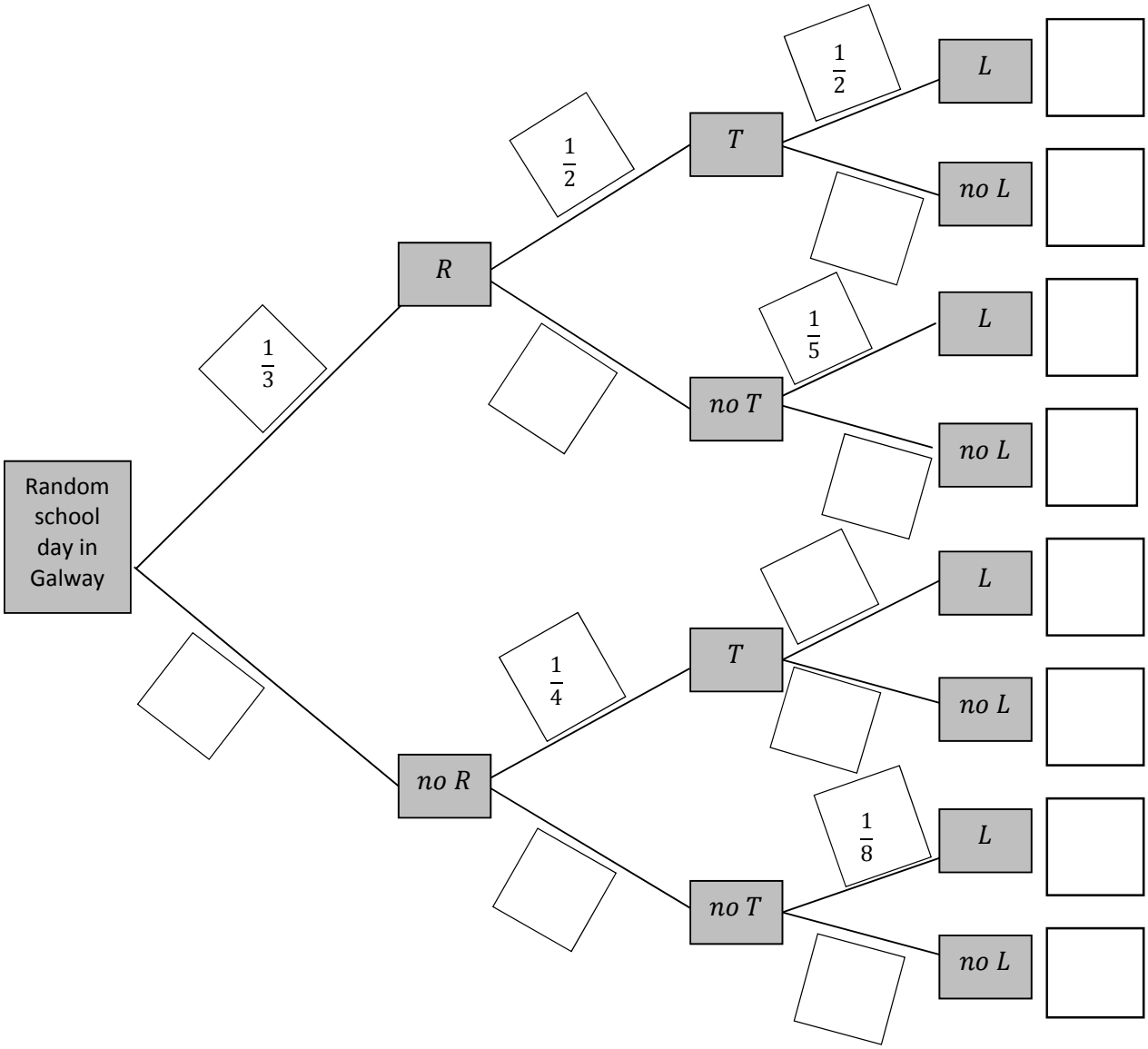
In any other situation the probability of being late for school is $\frac{1}{5}$.

Some of this information is shown in the tree diagram below.

(i) Write the probability associated with each branch of the tree diagram **and** the probability of each outcome into the blank boxes provided.

Give each answer in the form $\frac{a}{b}$, where $a, b \in \mathbb{N}$.

Key	Rain $= R$	Heavy traffic $= T$	Late $= L$
	No rain $= no\ R$	Not heavy traffic $= no\ T$	Not late $= no\ L$



(ii) On a random school day in Galway, find the probability of being late for school.

(iii) On a random school day in Galway, find the probability that it rained in the morning, given that you were late for school.

Question 9 **(50 marks)**

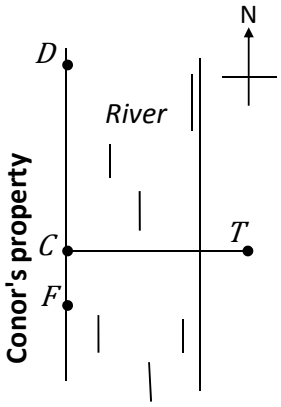


Figure 1

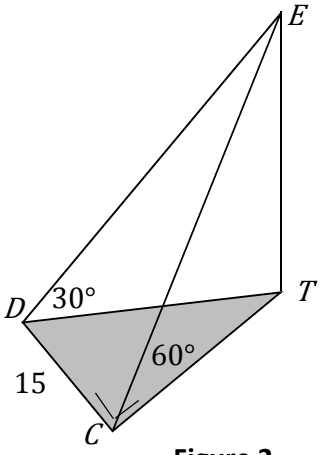


Figure 2

Conor's property is bounded by the straight bank of a river, as shown in **Figure 1** above.
T is the base of a vertical tree that is growing near the opposite bank of the river.
$|TE|$ is the height of the tree, as shown in **Figure 2** above.
From the point C, which is due west of the tree, the angle of elevation of E, the top of the tree, is $60°$.
From the point D, which is 15 m due north of C, the angle of elevation of E is $30°$ (see **Figure 2**).
The land on both sides of the river is flat and at the same level.

(a) Use triangle ECT, to express $|TE|$ in the form $\sqrt{a}|CT|$ metres, where $a \in \mathbb{N}$.

(b) Show that $|TE|$ may also be expressed as $\sqrt{\frac{225+|CT|^2}{3}}$ metres.

(c) Hence find $|CT|$, the distance from the base of the tree to the bank of the river at Conor's side. Give your answer correct to 1 decimal places.

(d) Find $|TE|$, the height of the tree. Give your answer correct to 1 decimal place.

(e) The tree falls across the river and hits the bank at Conor's side at the point F. Find the maximum size of the angle FTC. Give your answer in degrees, correct to 1 decimal place.

(f) If the tree was equally likely to fall in any direction, find the probability that it would hit the bank at Conor's side, when it falls.
Give your answer as a percentage, correct to 1 decimal place.

You may use this page for extra work.

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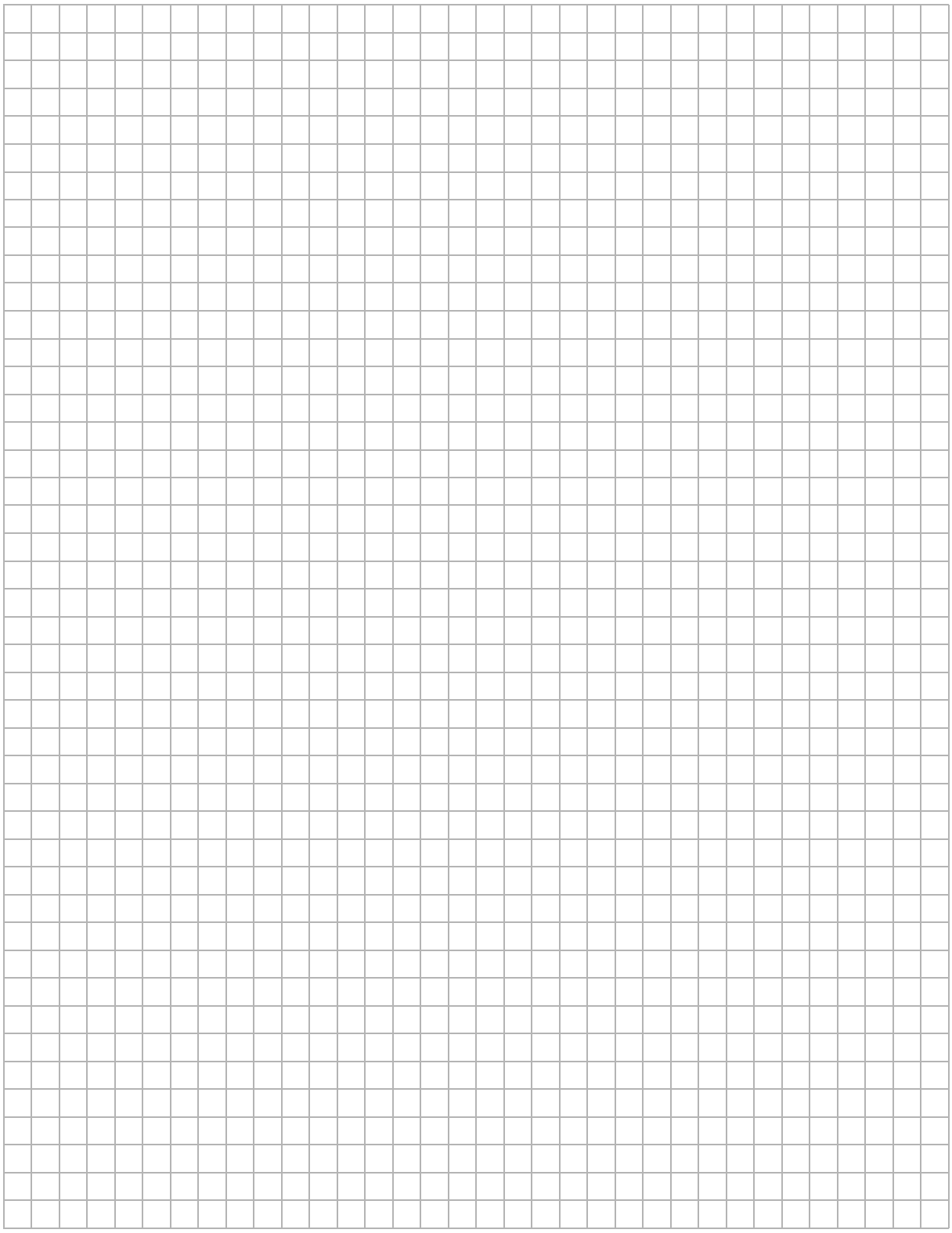

Leaving Certificate 2017 – Higher Level

Mathematics – Paper 2

Monday 12 June

Morning 9:30 – 12:00