Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION, 2009

MATHEMATICS – HIGHER LEVEL

PAPER 2 (300 marks)

MONDAY, 8 JUNE – MORNING, 9:30 to 12:00

Attempt **FIVE** questions from **Section A** and **ONE** question from **Section B**.
Each question carries 50 marks.

WARNING: **Marks will be lost if all necessary work is not clearly shown.**

Answers should include the appropriate units of measurement, where relevant.

SECTION A

Answer FIVE questions from this section.

1. **(a)** Show that, for all values of $t \in \mathbf{R}$, the point $\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right)$ lies on the circle $x^2 + y^2 = 1$.

(b) **(i)** Find the equation of the tangent to the circle $x^2 + y^2 = 10$ at the point $(3,1)$.

(ii) Find the values of $k \in \mathbf{R}$ for which the line $x - y + k = 0$ is a tangent to the circle $(x-3)^2 + (y+4)^2 = 50$.

(c) Two circles intersect at $p(2,0)$ and $q(-2,8)$. The distance from the centre of each circle to the common chord $[pq]$ is $\sqrt{20}$.

Find the equations of the two circles.

2. **(a)** If $\vec{a} = 2\vec{i} + \vec{j}$, $\vec{b} = -\vec{i} + 5\vec{j}$, find the unit vector in the direction of $\overrightarrow{ab}$.

(b) In the triangle *abc*, *p* is a point on the side $[bc]$.

The point *q* lies outside the triangle such that $\overrightarrow{pq} = \overrightarrow{pb} + \overrightarrow{pc} - \overrightarrow{pa}$.

(i) Express $\vec{q}$ in terms of $\vec{a}$, $\vec{b}$ and $\vec{c}$.

(ii) Hence show that *abqc* is a parallelogram.

(c) **(i)** $\vec{p} = 12\vec{i} + 5\vec{j}$ and $\vec{q} = 3\vec{i} + 4\vec{j}$.

Find the value of the scalar k such that

$$k\left|\vec{p}^{\perp} - \vec{q}\right| = \left|\vec{p}^{\perp}\right| - \left|\vec{q}\right|.$$

(ii) Prove that for all vectors $\vec{r}$ and $\vec{s}$

$$\left(\vec{r} - \vec{s}\right)^{\perp} = \vec{r}^{\perp} - \vec{s}^{\perp}.$$

3. **(a)** Find the equation of the line through the point $(1, 0)$ that also passes through the point of intersection of the lines $2x - y + 6 = 0$ and $10x + 3y - 2 = 0$.

(b) **(i)** Prove that the measure of one of the angles between two lines with slopes m_1 and m_2 is given by

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}.$$

(ii) Find the equations of the two lines that pass through the point $(6, 1)$ and make an angle of 45° with the line $x + 2y = 0$.

(c) f is the transformation $(x, y) \to (x', y')$, where $x' = -x + 2y$ and $y' = 2x - y$.

(i) L is the line $ax + by + c = 0$. Prove that $f(L)$ is a line.

(ii) The line $y = mx$ is its own image under f.
Find the two possible values of m.

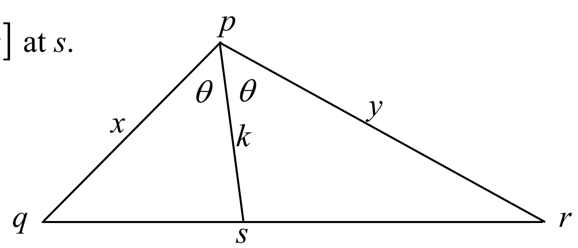
4. **(a)** Show that $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = 2$.

(b) The lengths of the sides of a triangle are 21, 17 and 10.
The smallest angle in the triangle is A.

(i) Show that $\cos A = \dfrac{15}{17}$.

(ii) Without evaluating A, find $\tan\dfrac{A}{2}$.

(c) The bisector of $\angle qpr$ meets $[qr]$ at s.
$|\angle qpr| = 2\theta$, $|pq| = x$,
$|pr| = y$ and $|ps| = k$.



(i) Find the area of the triangle pqs in terms of x, k and θ.

(ii) Show that $k = \dfrac{2xy\cos\theta}{x + y}$.

5. **(a)** Find all the solutions of the equation $\cos^2 x - \cos x = 0$, where $0° \le x \le 180°$.

(b) The function $f : x \to \sin^{-1} x$ is defined for $-1 \le x \le 1$.

(i) Copy and complete the table of values of f below.

x	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1
$f(x)$			$-\frac{\pi}{6}$				

(ii) Draw the graph of $y = f(x)$ on graph paper, noting that $\frac{\sqrt{3}}{2} \approx 0{\cdot}87$.

Scale the y-axis in terms of π.

(iii) State, with reason, whether each of the following statements is true.

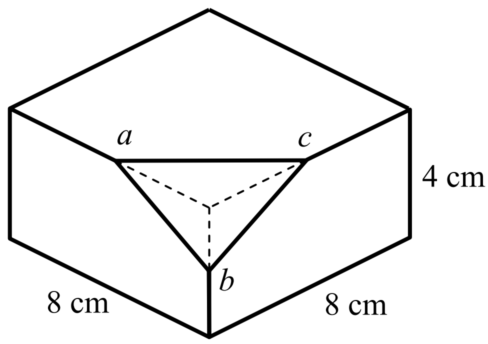
A: "If $\sin x_1 = \sin x_2$, then $x_1 = x_2$".

B: "If $\sin^{-1} x_1 = \sin^{-1} x_2$, then $x_1 = x_2$".

(c) A rectangular block of cheese measures 8 cm × 8 cm × 4 cm.

One corner is cut away from the block, in such a way that three of the edges are cut through their midpoints a, b and c.

Find the area of the triangular face abc created by the cut.



6. **(a)** A student taking a literature course has to read three novels from a list of ten novels.

(i) How many different selections of three novels are possible?

(ii) Two of the ten novels are by the same author. How many selections are possible if the student wishes to choose three novels by different authors?

(b) **(i)** In how many different ways can eight people be seated in a row?

(ii) Three girls and five boys sit in a row, arranged at random. Find the probability that the three girls are seated together.

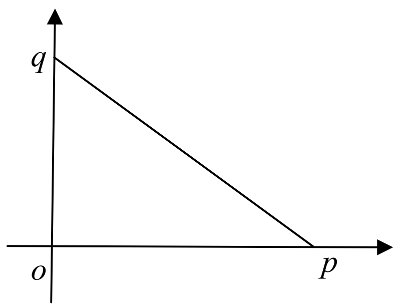
(iii) Three girls and n boys sit in a row, arranged at random. If the probability that the three girls are seated together is $\frac{1}{35}$, find the value of n.

(c) x and y are randomly selected integers with $1 \le x \le 10$ and $1 \le y \le 10$. p is the point with coordinates $(x, 0)$ and q is the point with coordinates $(0, y)$. Find the probability that

(i) the slope of pq is equal to -1

(ii) the slope of pq is greater than -1

(iii) the length of $[pq]$ is less than or equal to 5.



7 **(a)** The prices of four food items in a shopping basket are €3, €5, €1 and €6. Find the weighted mean price of these items using the weights 2, 3, 4 and 1 respectively.

(b) **(i)** Solve the difference equation $u_{n+2} - 6u_{n+1} + 5u_n = 0$, where $n \ge 1$, given that $u_1 = 0$ and $u_2 = 20$.

(ii) Find an expression in n for the sum of the terms $u_1 + u_2 + u_3 + ... + u_n$.

(c) The two numbers a and b have mean $\bar{x}$ and standard deviation σ_1. The three numbers c, d and e have mean $\bar{x}$ and standard deviation σ_2. Find the standard deviation of the five numbers a, b, c, d and e in terms of σ_1 and σ_2.

SECTION B

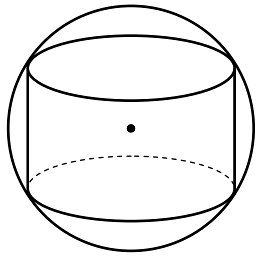
Answer ONE question from this section.

8. **(a)** Use integration by parts to find $\int xe^{4x}\,dx$.

(b) **(i)** Derive the first four terms of the Maclaurin series for $f(x) = \sqrt{1+x}$.

(ii) Given that this series converges for $-1 < x < 1$, use these four terms to find an approximation for $\sqrt{17}$, as a fraction.

(c) The diagram shows a cylinder inscribed in a sphere.
The cylinder has height $2x$ and radius r.
The sphere has fixed radius a.



(i) Express r in terms of a and x.

(ii) Find, in terms of a, the maximum possible volume of the cylinder.

9. **(a)** A and B are independent events such that $P(A) = 0{\cdot}25$ and $P(A \cup B) = 0{\cdot}55$.
Find $P(B)$.

(b) A person plays a game that involves throwing five hoops at a peg.
The following table gives the probability distribution for the number of hoops that land on the peg.

x	0	1	2	3	4	5
$P(x)$	0·01	0·08	0·23	0·34	0·26	0·08

Find the mean and the standard deviation of the distribution.

(c) A coin is slightly bent and is thought to favour heads. Accordingly, it is tossed 100 times to test the null hypothesis that it is fair against the alternative hypothesis that it favours heads. In this experiment, 55 heads are observed.

(i) Show that this result is not significant at the 5% level.

(ii) How many times would the coin have to be tossed in an experiment in order that an observation of 55% heads *would* be regarded as significant at the 5% level?

10. **(a)** If a is the permutation $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$, find $a \circ a$.

(b) The set $\{1, 2, 4, 5, 7, 8\}$ is a group under multiplication modulo 9.

(i) Draw up a Cayley table for the group.

(ii) Find a generator of the group.

(iii) Hence, or otherwise, find a subgroup of order 2 and a subgroup of order 3.

(c) $(G, \circ)$ and $(H, *)$ are two groups with identities e_G and e_H respectively.

If $\phi : G \to H$ is an isomorphism, prove that

(i) $\phi(e_G) = e_H$.

(ii) $\phi(x^{-1}) = [\phi(x)]^{-1}$, for all $x \in G$.

11. **(a)** Find the equation of the ellipse with foci $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$ and with eccentricity $\dfrac{\sqrt{7}}{4}$.

(b) A transformation f is a similarity transformation if there exists a fixed number k such that $|f(a)f(b)| = k|ab|$, for all a and b.
Show that angle measure is invariant under a similarity transformation.

(c) **(i)** Define the term *conjugate diameters* of an ellipse.

(ii) Prove that all parallelograms circumscribed to a given ellipse at the endpoints of conjugate diameters have the same area.

Blank Page